A FINITE ELEMENT PARAMETRIC STUDY FOR THE RESPONSE OF CONCRETE HIGHWAY PAVEMENTS WITH SKEWED JOINTS

By

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A FINITE ELEMENT PARAMETRIC STUDY FOR THE RESPONSE OF CONCRETE HIGHWAY PAVEMENTS WITH ECONOCRETE SUBBASES

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This research was performed to study an analytical behavior of skewed jointed concrete pavement with unbonded econocrete subbases. The major emphasis of the research was to provide a rational analysis approach, for evaluating the maximum stress levels to be expected in concrete pavements.

The first portion centered on improvements to two finite element analysis programs of pavement system, FEACONS and INPLANE. This consisted of derivations of finite element technique for the parallelogram plate bending element and in-plane stress element. Based on that, the program FEACONS and INPLANE were expanded to analyze the skewed jointed pavements and the perpendicular jointed pavements as well. Here the program FEACONS analyzes the effects of plate bending response of slabs and INPLANE evaluates the planar effects of slabs due to concrete shrinkage, thermal expansion

or contraction, and subgrade friction effects.

The second part of the study involved structural analysis of the pavements for various combinations of significant parameters, such as skewed angles, subgrade moduli, joint stiffnesses, temperature differentials, and axle load positions. As a result, a series of maximum stress tables were tabulated and even more stress and deflection profiles were produced. It certainly provided a concise way of looking at the distribution of stress and variations of displacements on the surface of concrete slab.

The results of parametric studies showed that the use of skewed joints induces slightly higher stresses, which should not in themselves affect structural performance. However, the higher stress are attained closer to the edge-corner, which may contribute to the higher frequency of corner cracking on the skewed jointed pavements. Besides, the use of higher subgrade modulus and higher joint stiffness does not affect maximum stress levels seriously but certainly produces different patterns of stress and deflection profiles, which are closely related to the current cracking pattern in concrete pavements.

Consequently, the highway engineers should consider the overall structural response of a slab rather than the magnitude of maximum failure stress. Those results might be applicable to an investigation of the premature cracking problems on concrete pavement.

CHAPTER 1
INTRODUCTION

In the early part of twentieth century, rigid pavements composed of a series of thin Portland Cement Concrete (PCC) slabs began to find widespread use in pavement construction because of desirable surface characteristics, durability and economy. However, the brittleness of concrete and its sensitivity to variations in moisture and temperature have sometimes caused serious distress and several types of failure mechanisms in these pavement.

The problems of distress and failures on some of Florida's newer concrete pavements, for instance, Interstate 75 (I-75) in Manatee and Sarasota counties, have been characterized by severe signs of pumping, faulting and cracking before the end of their intended service life. On this particular pavement, the joints are skewed with one foot per lane skewness (9.46^{o}) and doweled instead of using the customary normal transverse joints.

Traditionally, the design of concrete pavements has been based upon the assumption that the slab acts as a thin plate resting on an elastic subbase. The primary concerns considered in the design are bending stresses which occur mainly due to the live loads and temperature differentials.

Westergaard made the first serious attempt to find a theoretical solution for rigid pavement design in 1920. He made the the following important assumptions (Ref.1):

1. The concrete slab is a homogeneous elastic solid in equilibrium.
2. The reaction of the subgrade is proportional to the deflection of the slab.
3. The reaction of the subgrade per unit area at any given point is a function of a constant, Ks (modulus of subgrade reaction), and the deflection at that point.
4. The thickness of the concrete slab is uniform.
5. The load at the interior and at the corner of the slab is distributed uniformly over a circular contact area; for the corner loading, the circumference of this circular area is tangential to the edge of the slab.
6. The load at the interior edge of the slab is distributed uniformly over a semicircular contact area, the diameter of the semicircle being along the edge of the slab.

His analysis, however, neglects the stress induced by thermal and shrinkage strains. Although some treatment has been given to the bending effects of temperature differentials between the top and bottom of the slab, the inplane stresses caused by concrete shrinkage and by overall thermal expansion or contraction has largely been ignored.

In recent years, the appearance of high speed digital computers has made more accurate analysis possible,

specifically, analyses using the finite element method. A computer program, Finite Element Analysis of CONcrete Slabs (FEACONS III), was developed by Kevin Lee Toye and Shau Lei at the Department of Civil Engineering, University of Florida, in response to a need for a suitable analytical model to analyze the behavior of normal concrete pavements effectively and realistically. In addition, the computer program INPLANE was developed by Mark Landry at the Department of Civil Engineering, University of Florida, to perform a thorough analysis of the planar effects on a concrete pavement using the same nodal mesh as that for FEACONS.

Despite the improvement that FEACONS III and INPLANE represent over previous programs, there were certain limitations, in that those programs could not handle skewed joints. To enable FEACONS to properly account for skewed joints, the program was expanded to include a parallelogram -shaped element. The latest versions are called FEACONS V and INPLANE II. This work involved writing the coding for calculating the following matrices and vectors.

1) (12x12) Stiffness matrix of the parallelogram plate bending element.
2) (12x12) Stress matrix of the parallelogram plate bending element used to recover the stresses.
3) (12x1) Equivalent nodal load matrix due to uniformly distributed loads on the parallelogram element.

4) (12x1) Equivalent nodal load matrix due to thermal gradients on the parallelogram plate bending element.
5) (12x12) Stiffness matrix of the parallelogram in-plane stress element.
6) (12x12) Stress matrix of the parallelogram in-plane stress element.
7) (12x1) Equivalent nodal load matrix due to frictional resistance on the parallelogram in-plane stress element.
8) (12x1) Equivalent nodal load matrix due to concrete shrinkage on the parallelogram in-plane stress element.
9) (12x1) Equivalent nodal load matrix due to temperature variations on the parallelogram in-plane stress element.

Chapter III and IV consider the finite element derivations of parallelogram plate bending element and inplane stress element respectively.

The computer program FEACONS V can analyze a three-slab concrete pavement system, considering the effects of skewed joints, live loads, dead loads, subgrade voids, joint conditions and temperature variations within the concrete. Furthermore, INPLANE can now also compute in-plane stresses and displacements for pavements with skewed joints.

In spite of numerous recent improvements in the design and construction of pavements, some concrete pavements still exhibit premature cracking and other forms of distress, as mentioned previously. Therefore, it is necessary to thoroughly examine and evaluate some of the parameters that

can affect pavement performance. This highlights the need for better understanding of not only concrete pavement behavior, but also the parameters that influence such behavior. This research can eventually lead to an improved design procedure for concrete pavements. More specifically, this research should contribute to effective remedies to the econocrete pavement behavior problem, by giving a better understanding to pavement engineers concerning how the various parameters involved in concrete pavement behavior affect performance and how those factors interact.

CHAPTER 2
LITERATURE REVIEW AND BACKGROUND

2.1 Concepts of Rigid Pavement

In general, pavements may be classified as two types: rigid and flexible. As commonly used, the term "rigid pavement" is applied to wearing surfaces constructed of Portland cement concrete. The concrete pavement, properly designed and constructed, possesses structural strength, desirable surface characteristics, durability and economy.

An outstanding property of the concrete pavement as shown in Figure 2.1, 2.2, and 2.3 is, no doubt, its prolonged durability when continuously subjected to the most severe traffic and climatic conditions. With respect to structural strength, the concrete pavement can be designed to meet practically any requirement by thickening certain sections of the slab which may be subjected to more severe stresses than other parts, although this is seldom done, and by limiting the horizontal dimensions of slab units through the use of joints.

Also, effective protection against the detrimental effects of unpredictable conditions can be provided by the use of steel reinforcement--a procedure which is impossible with any other type of pavement, since concrete is the only

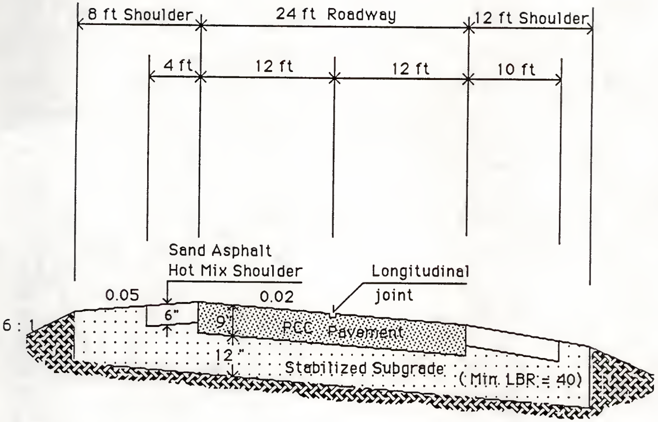


FIGURE 2.1 TYPICAL PAVEMENT SECTION OF I-10 CONCRETE PAVEMENT USING A CONVENTIONAL STABILIZED SUBGRADE

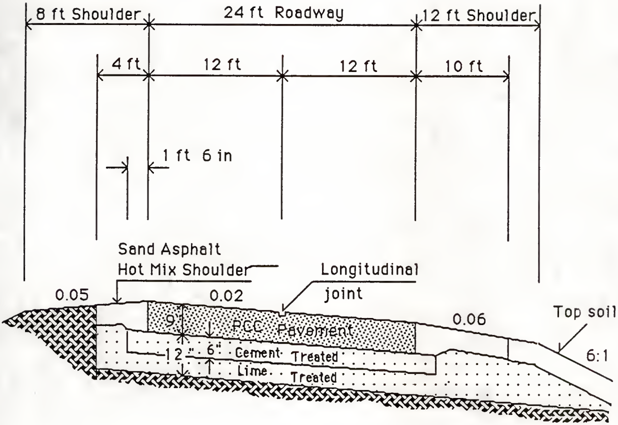


FIGURE 2.2 TYPICAL PAVEMENT SECTION OF I-10 CONCRETE PAVEMENT USING A LIME AND CEMENT TREATED SUBBASE

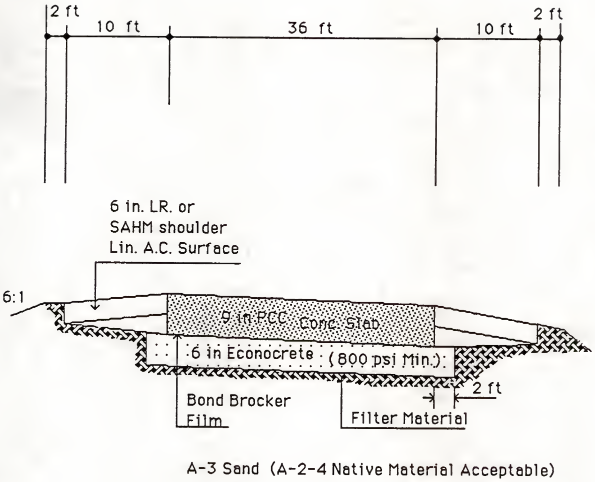


FIGURE 2.3 TYPICAL PAVEMENT SECTION OF I-75 CONCRETE PAVEMENT IN SARASOTA AND MANATEE COUNTIES

paving material that will bond with steel in such a manner as to effectively conserve the tensile resisting capacity of the slab section even though the concrete itself may rupture. Furthermore, while comparing favorably with other forms of construction in initial cost, the real economy of the concrete pavement is embodied in its long life and low cost of maintenance.

Besides, an FDOT (Florida Department of Transportation) report lists some supporting reasons for the use of concrete pavement for portions of I-75. These include (Ref.2):

(1) Concrete pavements had relatively good performance records in comparison with a list of 40 concrete paving projects in all parts of Florida which were 20 years old or older and still were in good or excellent condition.

(2) Concrete pavements could be less expensive to maintain, as compared to asphalt pavements.

(3) Concrete pavements did not have to be repaired or maintained as often and thus the interruption of traffic could be minimized.

(4) In urban areas where there were numerous concrete bridges for overpasses and interchanges, concrete pavements would provide continuity of pavement surface type.

(5) Flexible pavements tended to lose their skin resistance at a rate higher than that of concrete pavements. Thus, it would be safer to drive on a concrete pavement than on an asphalt pavement of the same age.

2.2 Function of Skewed Joints in Concrete Pavements.

During the 1920s, the early concrete pavements were constructed without longitudinal or transverse joints. Subsequently, the majority of these pavements developed longitudinal cracks that led to serious pumping, cracking, and faulting problems. In addition to longitudinal cracks, transverse cracks occurred at variable spacings shortly after construction. Therefore, expansion, contraction, and longitudinal joints were introduced in an attempt to control the pavement cracks and minimize pavement distress. The primary function of expansion joints is to prevent the development of damaging compressive stresses due to volume increase in the pavement slabs, and to avoid excessive pressures from being transmitted to adjacent structures. The function of contraction joints is to provide for an orderly arrangement of transverse cracking in the pavement when the concrete shrinks.

In the recent years, highway engineers introduced skewed joints in concrete pavements with a 4 foot to 5 foot skew on a 24 foot width pavement, which results in only one wheel crossing the joint at any one time and provides better riding quality across the joint.

2.3 Review of Existing Computer Models.

2.3.1 Introduction

Several computer models for analyzing concrete pavements have been developed since the 1960s. The principal analysis programs are:

(1) ILLISLAB by Tabatabaie and Barenberg (Ref.3)

(2) JSLAB by Tabatabaie and Colley (Ref.4)

(3) WESLIQUID and WESLAYER by the U.S.Army Engineer Waterways Experiment Station. The finite element method for modeling concrete pavements generally utilizes the thin plate theory. The concrete pavement system is divided into the following finite elements: rectangular elements for the slab, spring elements for the subgrade, linear and rotational springs for joints to model the aggregate interlock, keyway and dowel bars. The foundations as a base or a subbase are idealized either as a Winkler (dense liquid) foundation or as an elastic solid foundation. A Winkler foundation can be visualized as a bed of springs with each spring deforming independently. The stiffness of Winkler springs represents the foundation stiffness, which produces a reaction that is equal to the deflection multiplied by the spring stiffness. The elastic solid foundation idealizes the foundation as a homogeneous, elastic, and isotropic solid having a semi-infinite depth, characterized by a modulus of elasticity (E) and Poisson's ratio. The assumptions and modeling

techniques for those computer programs are briefly illustrated in this chapter.

2.3.2. WESLIQUID and WESLAYER

Both of these finite element models are based on the classical theory of thin plates with small deformations. The slab is modeled using the rectangular plate element developed by Zienkiewicz and Cheung (Ref.5). The basic difference between the models is in their modeling of the sublayers, which are considered as an elastic layered solid in the WESLAYER model and as a Winkler foundation in the WESLIQUID model.

In the WESLIQUID model, the sublayers are considered as a dense liquid material modeled by a series of uniformly distributed elastic springs. The forces due to the reaction at the sublayers are simply added to the forces of the plate at any given node. These reactions have only a vertical component which is added to the vertical component of the nodal forces in the plate (Ref.6).

In the WESLAYER model, the sublayers are considered as a layered elastic solid, in which the deflection at any given point depends, not only on the forces at the node, but also on the forces and deflections at other nodes. The stiffness matrix for the sublayers is obtained by inverting the flexibility matrix formed using the Burmister equation.

The WESLIQUID and WESLAYER models can consider the effect of a linear temperature gradient in the concrete slab.

The initial deflection due to the temperature gradient is computed using the coefficient of thermal expansion, the temperature gradient, the thickness, and the distance to the center of the slab where the deflection is zero. The equilibrium equations for the nodes are formed including the temperature effects. Full contact or partial contact between slab and sublayer can be assumed.

The analysis of the joints is accomplished by combining different amounts of shear and moment transfers. The two models have three options for specifying shear transfer and one for moment. Specification of shear transfer can be accomplished by a shear transfer efficiency factor, defined by the ratio of the vertical deflections between loaded and unloaded slabs along the joint; a spring constant modeled by imaginary springs along the joints; and by diameter and spacing of dowel bars, which is applicable only when steel bars are the only means of shear transfer. It is also possible to specify the conditions of looseness of the dowel bars by a dowel support factor.

Moment transfer across joints or cracks is accomplished in a two step procedure. First, the moments at nodes along the crack or joint are calculated assuming a perfect rigid joint. Then these moments are multipied by the efficiency of moment transfer, and assigned to each slab as externally applied moments. In a second step the applied moments are included in the analysis.

2.3.3. ILLISLAB

In this model, the subgrade is considered as a Winkler type as in WESLIQUID. It behaves as a dense liquid for which the deformation at a certain point depends only on the stress at that point. The stress deformation relation in the subgrade is represented by k, the modulus of subgrade reaction. The values for this parameter can be varied from node to node and can be taken as zero at points with no support.

The dowel bars behave as linearly elastic bars located at the neutral axis of the plates. These bars transfer shear force and bending moment from the slab on one side of the joint to the slab on the opposite side.

The Winkler foundation is modeled by a series of spring elements distributed uniformly under the slab. The spring elements have one degree of freedom per node: a vertical displacement. Corresponding to this, there is a vertical force. The force at each element is given by the deformation of the element times the spring constant. This spring constant is the modulus of subgrade reaction for the subgrade at the location of the element. The value for this parameter can be changed from node to node.

Dowels are modeled by bars with two degrees of freedom per node. These degrees of freedom are a vertical displacement and a rotation around a horizontal transverse

axis. Therefore, the bar is able to transfer a vertical shear force and a moment. In addition, some looseness between the dowel bar and the surrounding concrete is modeled by a spring element whose constant is the so called concrete dowel interaction factor.

Aggregate interlock and keyway joints are modeled by spring elements having a vertical displacement as a single degree of freedom. Vertical forces are transferred across the joints by the springs.

2.3.4 JSLAB

In addition, JSLAB developed by the Portland Cement Association has similar modeling capabilities as the WESLIQUID models on Winkler foundation. Tayabji and Colley utilized this JSLAB to evaluate the effect of joints with non-uniformly spaced dowels (Ref.4).

2.4. Failure Mechanisms in Concrete Pavement.

A concrete pavement which has shown severe premature has been viewed with great concern by highway officials and engineers. As traffic loads are applied, a pavement system experiences deformation, stress, strain, or surface wearing in the presence of thermal gradients and moisture variation. Distress usually occurs when pavement response exceeds the allowable values of strength and durability with heavy traffic loads and extreme temperatures.

Major types of pavement distress are listed in Table 2.1 which also includes severity, description and mechanism of each distress. Figures 2.4 and 2.5 present schematic diagrams of the major types of pavement distress. It is evident that most types of pavement distress are either developed at the joints or associated with joint behavior.

A report on joint-related distress(Ref.3) considered heavy traffic loads and extreme temperatures as responsible for initiating pavement distress. However, stresses generated by load and/or temperature usually do not result in pavement distress unless coupled with stresses caused by other deficiencies. The report cited many pavement design and construction deficiencies as the possible contributors. Some of the deficiencies listed were excessive slab length, narrow pavement, inadequate load transfer, misalignment of dowels, improper joint construction, infiltration of incompressibles and pumping.

It is important to note that pumping is not a stress initiator but can cause other serious forms of distress that can ultimately lead to pavement failure. In fact, pumping is generally recognized as the major problem in jointed concrete pavements. Haveem (Ref.7) viewed pumping as a phenomenon which indicates imminent distress or failure of a pavement. He presented an accurate description of the events leading to the pumping phenomena. The upward slab curling

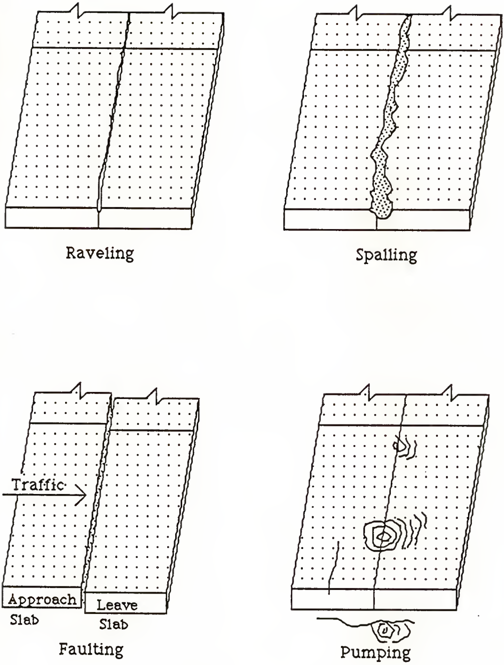


FIGURE 2.4 TYPES OF JOINT DISTRESS (1)

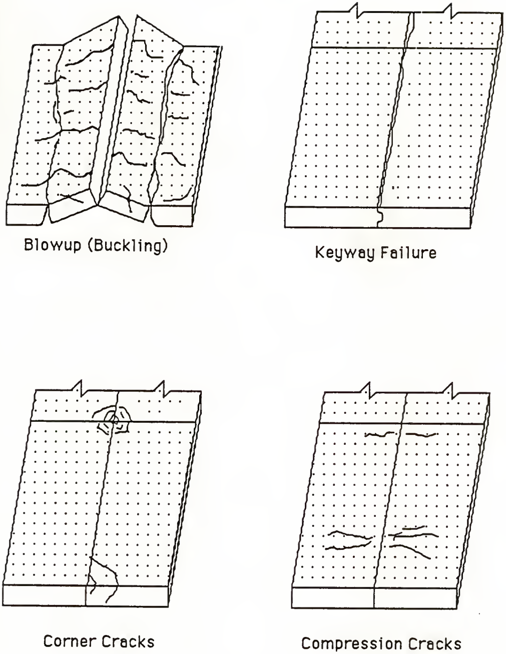


FIGURE 2.5 TYPES OF JOINT DISTRESS (2)

Table 2.1 CONCRETE PAVEMENT DISTRESS (Ref.8)

Type 1 : Pumping

Severity	High to very high
Description	Ejection of mixtures of water and fine subgrade materials through joints or crack.
Mechanism	Free water in supporting layer squeezed out under traffic load carry out fines from beneath the slab.

Type 2 : Faulting

Severity	High to very high
Description	Differential vertical displacement of adjoining slab, creating a "step" deformation in the pavement surface
Mechanism	Differential settlement or swelling between two adjacent slabs, or uneven support under the slabs associated with pumping

Type 3 : Transverse Cracking

Severity	High to very high
Description	A crack or break in the middle third of the slab at right angles to the pavement centerline
Mechanism	Insufficient contraction joints; overloading an upward curled slab having inadequate subgrade support

Type 4 : Corner Cracking

Severity	High
Description	Diagonal crack extending from the joint to edge of the pavement slab
Mechanism	Poor subgrade support at slab corner associated with pumping and/or excessive upward curling; or overall weakness in subgrade support

Type 5 : Longitudinal Cracking

Severity	High
Description	A crack or break approximately parallel to the pavement centerline
Mechanism	Lateral shrinkage; lateral movement and loss of subgrade support; possible bending or curling

Table 2.1 Continued

Type 6 :	Spalling	
	Severity	High
	Description	Breakdown of slab at joints or cracks, resulting in the removal of sound concrete
	Mechanism	Improper joint construction; incompressibles in joint; dowel misalignment
Type 7 :	Buckling Blowup	
	Severity	Very high
	Description	Lateral break up of concrete near the joint with two sides of joint lifted off the subgrade
	Mechanism	Joint lockup due to infiltration of incompressible, resulting in excessive bending stresses
Type 8 :	Surface Deterioration (scaling, raveling)	
	Severity	Moderate
	Description	Progressive disintegration and loss of the concrete wearing surface
	Mechanism	Concrete surface erosion by de-icing chemicals; improper construction techniques; repeatitive freeze-thaw cycles

and/or warping in the vicinity of a joint creates a void beneath the slab. Water infiltrating the pavement through the joint accumulates in the void. The entrapped water is subjected to considerable pressure with each passage of heavy axle loads. Water flows laterally at a high velocity forcing the removal of fine materials from the subgrade, through the joint, and along the edge of slab. As pumping action continues, a void will be created beneath the joint. Loss of subgrade support at the joint combined with excessive upward curling and heavy axle loads can induce stresses high enough to cause corner cracking.

2.5 Concrete Pavement Design Guide

The theories of design of concrete pavements are different from those of flexible pavements. While a flexible type pavement basically distributes the load gradually to the layers underneath, a rigid pavement acts as a structural element (a plate) resting on a foundation. Since Westergaard's work (Ref.1) in the 1920s, many design guides and procedures for concrete pavement have been developed by highway officials and private agencies. One of these is the AASHTO (American Association of State Highway and Transportation Officials) design guide, while another is the PCA (Portland Cement Association) thickness design method.

2.5.1 The AASHTO Design Guide

In 1972, AASHTO published the " AASHTO Interim Guide for Design of Pavement Structures" (Ref. 9) based on the results from the AASHO Road Test (Ref. 10) and Spangler's (Ref.11) corner equation. The design procedure includes the determination of the thickness of the Portland cement concrete pavement slab, and the design of joints and of steel reinforcement. Also included are recommendations as to the treatment of subbase soils and the type and thickness of subbase required.

A new proposed AASHTO guide was published in 1986 (Ref.12). Some of the new concepts and parameters introduced

into the basic AASHTO design are

1. Reliability factor: to account for shift in the design traffic.
2. Environment: adjustment of design for parameters such as freeze and thaw, and swelling soils.
3. Drainage: to provide guidance in the design of sub-surface drainage systems, and modify design equations to take advantage of improvements in pavement performance associated with good drainage.
4. Loss of foundation support: to account for possible soil erosion under rigid pavements, especially at joints.
5. Effect of type of load transfer at joints: incorporating load transfer coefficients based on type of joint, and support conditions from the boundaries.
6. Effect of concrete shoulders: a procedure is provided for the design of rigid pavements with tied shoulders or a widened outside lanes.

2.5.2 The PCA Thickness Design

The general objective of the PCA design procedure is to determine the minimum thickness that will give the least annual cost. The PCA recommends two design criteria (Ref.13):

1. Fatigue, to keep pavement stresses caused by repeated loads within acceptable limits to prevent fatigue cracking.
2. Erosion, to limit the effects of pavement deflections of joints, corners, and the edges of slabs in order to control the erosion of foundation materials.

Concrete is subject to fatigue, as are other construction materials. In the PCA design concept, a fatigue failure occurs when a material ruptures under continued repetition of loads that cause stress ratios of less than 1. The stress ratio is the ratio of flexural stress to the modulus of rupture.

Flexural fatigue research on concrete has shown that the number of stress repetitions to failure increases as the stress ratio decreases; studies show that, if the stress ratio is less than 0.55, concrete will withstand virtually unlimited stress repetitions without loss in load-bearing capacity. To be conservative, designers reduce this ratio to 0.50.

Procedures recommended by the PCA are based on a design period of 20 years. The PCA design procedure takes into account the effects of the erosion of foundation and shoulder materials due to pavement deflections at slab edges, joints, and corners. This analysis recognizes modes of pavement distress such as pumping, faulting, and shoulder problems that are unrelated to fatigue. Another major factor in designing a concrete pavement is, of course, the traffic to which it will be subjected during its service life. The number and magnitude of heavy vehicle loads are critical. The PCA manual details two methods for determining traffic volumes and the number and magnitude of heavy wheel load repetitions.

2.5.3 The ACI Design Method

The ACI committee 325 (Ref.14) publishes a design method to determine the pavement thickness and amount of reinforcing steel for continuously reinforced concrete pavement. This method is based on some modifications of the AASHTO method. In continuously reinforced concrete pavements, the slab must resist stresses and deflections produced by applied traffic loads. The function of the longitudinal steel is to keep the cracks in the concrete tightly closed so that there is effective load transfer across the cracks. This longitidinal steel is provided to control cracking due to shrinkage and temperature. The results obtained with both methods are comparable, and the difference depends on the value selected for the load transfer factor. The ACI committee 325 recommends a value of 2.2 for the load transfer factor. Assuming this value, the results obtained with the AASHTO and the ACI methods are similar.

CHAPTER 3
FINITE PLATE BENDING ELEMENT

3.1 Introduction.

The finite element formulation of the parallelogram plate bending element is presented here. The element is based on the MZC rectangular plate bending element as developed by Melosh, Zienkiewicz and Cheung (Ref.5).

The basic assumptions are the classical thin plate theory which assumes that

(1) Thickness of the plate is small compared to its length and width.

(2) Lateral bending displacements are small compared to the thickness of the plate.

(3) Plane sections remains plane after deformation.

3.2 Parallelogram Plate Bending Element.

3.2.1 Introduction

The element used in this analysis is the four node parallelogram plate bending element. The material that comprises the element is assumed to be linear, elastic, homogeneous, and isotropic.

A parallelogram plate bending element shown in Figure 3.1 has three independent displacements at each node,

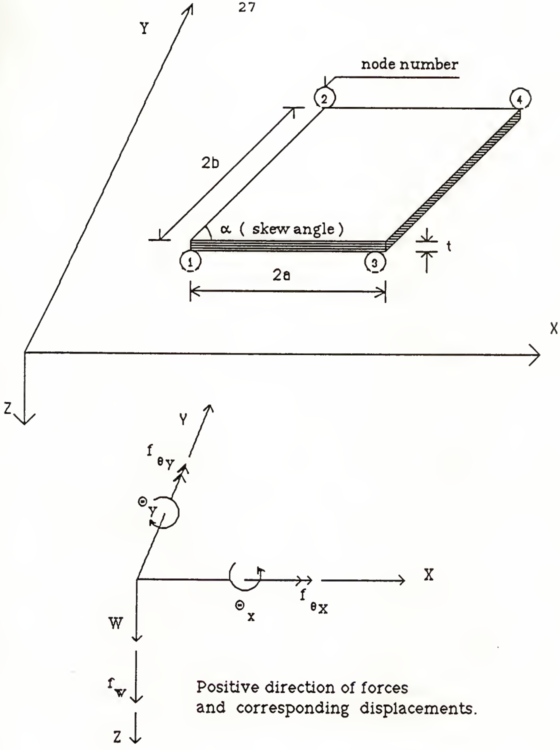


FIGURE 3.1 DIMENSIONS AND POSITIVE DIRECTION OF A PARALLELOGRAM PLATE BENDING ELEMENT

which are a vertical deflection, w, and rotations about the X and Y axes, for a total of twelve degrees of freedom per element.

The displacements shown in the Figure 3.1 correspond to the assumed positive directions of nodal forces at each node. The array of nodal displacements at the i^{th} node is

$$q_i = \begin{bmatrix} q_i1 \\ q_i2 \\ q_i3 \end{bmatrix} = \begin{bmatrix} wi \\ rxi \\ ryi \end{bmatrix} = \begin{bmatrix} wi \\ -dwi/dy \\ dwi/dx \end{bmatrix}$$

In the above vectors i can range from 1 to 4 for the four nodes. The array of corresponding nodal forces is

$$p_i = \begin{bmatrix} p_i1 \\ p_i2 \\ p_i3 \end{bmatrix} = \begin{bmatrix} Fz_i \\ Mx_i \\ My_i \end{bmatrix}$$

3.2.2 Coordinate Mapping.

Because of the parallelogram shape of the element, it is most efficient to convert the usual perpendicular (X,Y) coordinate system into an inclined (R,S) system, as shown in Figure 3.2, for formulation of the stiffness matrix and load vectors. Furthermore, if the Gauss Quadrature method is to be used to perform the numerical integrations involved in matrix formulation, these inclined (R,S) coordinates must also be normalized such that the element is enclosed in the range (-1, 1). The basic relationships between the two coordinate systems shown in Figure 3.2 are

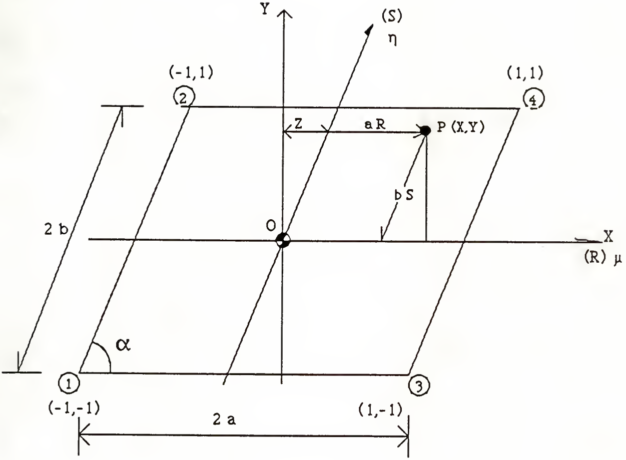


FIGURE 3.2 COORDINATE MAPPING OF PARALLELOGRAM ELEMENT

$$X = aR + Z$$

$$Y = bS\ Sin(\alpha)$$

Eliminating Z,

$$Tan(\alpha) = Y/Z,$$

$$Z = Y\ Cot(\alpha)$$

$$= bS\ Sin(\alpha)\ Cot(\alpha)$$

$$= bS\ Cos(\alpha)$$

Therefore, $X = aR + bS\ Cos(\alpha)$

$$Y = bS\ Sin(\alpha)$$

Or, $R = (X - Y\ Cot(\alpha))/a$

$$S = Y\ Cosec(\alpha)/b$$

The chain rule of differentiation of a general function of R and S, f(R,S), with respect to X and Y leads to

$$df(R,S)/dx = df/dR\ dR/dx + df/dS\ dS/dX$$

$$= df(R,S)/dR\ 1/a$$

$$df(R,S)/dY = df/dR\ dR/dY + df/dS\ dS/dY$$

$$= df(R,S)/dR\ (-Cot(\alpha)/a)$$

$$+ df(R,S)/dS\ (Cosec(\alpha)/b)$$

3.3 Assumed Displacement Function.

The generic displacement function, vertical displacement w, within an element expressed in terms of X,Y or R,S coordinates is

$$w = C_1 + C_2R + C_3S + C_4R^2 + C_5RS + C_6S^2 + C_7R3 + C_8R^2S + C_9RS^2 + C_{10}S^3 + C_{11}R^3S + C_{12}RS^3$$

The above expression is a complete cubic of ten terms and two

quartic terms. From this assumption it is possible to derive the displacement shape functions (f_i), which relate nodal displacements(q_i) at the ith node to generic displacements,

$$f_i = [\ f_i1\ ,\ f_i2\ ,\ f_i3\ ]$$

corresponding to,

$$q_i = [\ w_i\ ,\ rx_i\ ,\ ry_i\ ]\ .$$

Refer to Table 3.1 for the displacement shape function.

Table 3.1 : DISPLACEMENT SHAPE FUNCTION

Node #	Coordinate R , S	Degree of Freedom #	Shape Function.
		$f_i1=1$	$f1=(1-R)(1-S)(2-R-S-R^2-S^2)/8$
1	R=-1, S=-1	$f_i2=2$	$f2=-b(1-R)(1+S)(1-S)^2/8$
		$f_i3=3$	$f3=a(1+R)(1-S)(1-R)^2/8$
		$f_i1=4$	$f4=(1-R)(1+S)(2-R+S-R^2-S^2)/8$
2	R=-1, S=1	$f_i2=5$	$f5=b(1-R)(1-S)(1+S)^2/8$
		$f_i3=6$	$f6=a(1+R)(1+S)(1-R)^2/8$
		$f_i1=7$	$f7=(1+R)(1-S)(2+R-S-R^2-S^2)/8$
3	R=1, S=-1	$f_i2=8$	$f8=-b(1+R)(1+S)(1-S)^2/8$
		$f_i3=9$	$f9=-a(1-R)(1-S)(1+R)^2/8$
		$f_i1=10$	$f10=(1+R)(1+S)(2+R+S-R^2-S^2)/8$
4	R=1, S=1	$f_i2=11$	$f11=b(1+R)(1-S)(1+S)^2/8$
		$f_i3=12$	$f12=-a(1-R)(1+S)(1+R)^2/8$

The shape functions, which now can be considered simply as the displacement $w = f(X,Y)$ within an element must be definable in terms of q_i, the displacements at the i^{th} node.

Let

$$w = C_1 + C_2X + C_3Y + C_4X^2 + C_5XY + C_6Y^2 + C_7X^3 + C_8X^2Y + C_9XY^2 + C_{10}Y^3 + C_{11}X^3Y + C_{12}XY^3$$

$$(-dw/dy)_i = rX_i = -C_3 + C_5X + \ldots + \text{etc}$$

$$(dw/dx)_i = rY_i = C_2 + 2C_4X + \ldots + \text{etc}$$

where the coordinates of the i^{th} node are substituted for X and Y. Listing all twelve equations we can write, in matrix form,

$$\{ q_i \} = \begin{bmatrix} w_i \\ rx_i \\ ry_i \end{bmatrix} = [C] \{ V \}$$

Where [C] is a twelve-by-twelve matrix which is a function of nodal coordinates, and { V } is a vector of the twelve unknown constants. Inverting

$$\{ V \} = [C]^{-1} \{ qi \}$$

substituting $w = [g][V]$

where $[g] = \{ 1, X, Y, X^2, XY, Y^2 \ldots \}$ 1x12 a vector,

Now, $w = [g][C]^{-1}\{ q_i \}$

$= \{ f \}\{ q_i \}$

In the above, { f } includes a twelve-shape functions in X and Y, one for each of the twelve degrees of freedom. Displacement shape functions are given in Table 3.1.

3.4 Virtual Work Basis of Finite Element Methods.

We begin with definitions and invoke the virtual work principle for a general element. It will first be necessary to formulate the strains in terms of displacements. Let the generic displacements at any point within the element be expressed as the column vector U = { u , v , w } and nodal displacements {q} = { q_i } where q_i = { qx_i, qy_i, qz_i } for the i^{th} node and {q} is a 12 x 1 vector.

3.4.1 Displacement-Strain-Stress Relationships.

Displacement shape functions {f} relate generic displacements (U) to nodal displacements {q}, where {q} is the 12 x 1 vector including all degrees of freedom.

$$U = \{ f \}\{ q \} \qquad (3.1)$$

Strain can be obtained by differentiation of the generic displacements(U),

$$\{ \epsilon \} = d\ U. \qquad (3.2)$$

where d is the general differential operator.

Therefore,

$$\epsilon x = dU/dx,\ \epsilon y = dV/dy,\ \epsilon xy = dU/dy + dV/dx$$

Substituting (3.1) to (3.2),

$$\{ \epsilon \} = d\ U = d \{ f \}\{ q \} = B \{ q \} \qquad (3.3)$$

In the above, B is the Strain-Displacement operator (B = d {f}), and relates nodal displacements at each point to strain matrix and can be obtained by the differentiation of shape functions.

Simple continuum mechanics shows

$$\epsilon = [E]\ (\epsilon - \epsilon_o) \qquad (3.4)$$

Substituting (3.3) to (3.4),

$$\epsilon = [E]\ \{(Bq) - \epsilon_o\} \qquad (3.5)$$

in which the matrix product E B , which is called stress matrix, gives stresses at a generic point.

3.4.2 Virtual Work Principle.

By definition, the virtual work of external actions is equal to the virtual strain energy of internal stresses.

$$dU_i = dW_e \qquad (3.6)$$

Assume a set of nodal virtual displacements, $\{q^v\}$. The resulting displacements and strains within the element are

$$[\ U^v\ ] = [\ f\ ]\ \{\ q^v\ \}$$

$$[\ \epsilon^v\ ] = [\ B\ ]\ \{\ q^v\ \}$$

The work done by the nodal forces is equal to the sum of the products of the individual force components and the corresponding displacements.

dW_e = the virtual work of external forces.

$= dW_{ec} + dW_{ed}$

In the above, dW_{ec} and dW_{ed} represent the works due to concentrated and distributed loads, respectively. These are given by

$$dW_{ec} = \{\ q^v\ \}\ \{F\ \}$$

$$dW_{ed} = \int [U^v]^T\ w\ dV$$

$$dW_{ed} = \{q^V\}^T \cdot \int_v [\sigma]^T \; w \; dV$$

where w = distributed load

Similarly, the internal work per unit volume done by the stresses dU_i is given by

$$dUi = \int_v [\epsilon^V]^T \{ \sigma \} \; dV$$
$$= \{q^V\}^T \int_v [B]^T \{\sigma\} \; dV$$

Equating the external work with the total internal work and integrating over the volume of the element,

$$\{q^V\}^T \{F\} = \{q^V\}^T \left(\int [B]^T \{\sigma\} \; dV - \int [f]^T \; w \; dV \right)$$

Substituting Eqs.(3.1) through (3.5),

$$\{F\} = \left(\int [B]^T [E] [B] \; dV \right) \{q\} - \int [B]^T [E] \{\epsilon_0\} \; dV - \int [f]^T \; w \; dV \qquad (3.7)$$

Equation 3.7 is a basic equilibrium equation characteristic of any structural analysis problem. Its terms represent basic structural parameters. The stiffness matrix, [K], is given by

$$[\, K \,] = \int [B]^T [E] [B] \; dV \qquad (3.8)$$

Nodal forces due to distributed loads are

$$\{ Fw \} = - \int [f]^T \; w \; dV \qquad (3.9)$$

Nodal forces due to initial strains are

$$\{ F_{\epsilon 0} \} = - \int [B]^T [E] \{\epsilon_0\} \; dV \qquad (3.10)$$

3.5 Stiffness Matrix

As defined earlier, B operator relates nodal displacements {q} to the strain matrix $[\epsilon]$. From equation 3.2

strains can be obtained by differentiation of the generic displacements. From the basic geometry of displacement and deformation,

$$u = -z\, dw/dx$$

$$v = -z\, dw/dy \qquad (3.11)$$

Substituting equation (3.11) to (3.2),

$$\epsilon = d\, u \qquad (3.2)$$

$$\epsilon x = du/dx = -z\, d^2w/dx^2$$

$$\epsilon y = dv/dy = -z\, d^2w/dy^2$$

$$\epsilon xy = du/dy + dv/dx = 2z\, d^2w/dxdy$$

Therefore, we can express the differential operator d as

$$d = \{ -d^2w/dx^2, -d^2w/dy^2, 2d^2w/dxdy \}$$

From equation (3.3), the B operator can be obtained by differentiation of the shape functions, which were defined in the previous section. Partitioning [B] for the four nodes, we obtain $[B_i]$ and 3 x 3 matrix for the i^{th} node.

$$B_i = d\, f = \begin{bmatrix} -d^2/dx^2 \\ -d^2/dy^2 \\ 2d^2/dxdy \end{bmatrix} \{ f_i1, f_i2, f_i3 \}$$

$$B_i = \begin{bmatrix} -f_i1,xx & -f_i2,xx & -f_i3,xx \\ -f_i1,yy & -f_i2,yy & -f_i3,yy \\ 2f_i1,xy & 2f_i2,xy & 2f_i3,xy \end{bmatrix} \qquad (3.12)$$

To differentiate the displacement shape functions, which are expressed in terms of the dimensionless coordinates

R,S with respect to X,Y, we must use the chain rule for partial derivatives as shown in the Section 2.1. This results in the operator B expressed in terms of R and S.

From equation (3.9)

$$K = \int B^T \, E \, B \, dV \qquad (\text{ where } dV = dXdYdZ \text{ })$$
$$= t \, [J] \iint B^T \, E \, B \, dRdS \qquad (\text{ where } -1 \text{ to } 1 \text{ integration})$$

The Jacobian matrix, [J], in the above expression can be expressed as follows, using the chain rule.

$$\begin{bmatrix} df/dR \\ df/dS \end{bmatrix} = \begin{bmatrix} dX/dR & dY/dR \\ dX/dS & dY/dS \end{bmatrix} \cdot \begin{bmatrix} df/dX \\ df/dY \end{bmatrix}$$

$$\text{Jacobian} \quad [J] = \begin{bmatrix} dX/dR & dY/dR \\ dX/dS & dY/dS \end{bmatrix} = ab \, \text{Sin}(\alpha)$$

Therefore, the stiffness matrix K becomes

$$K = t \, ab \, \text{Sin}(\alpha) \iint B^T \, E \, B \, dRdS \qquad (3.13)$$

After finding the B operator by the second differentiation of the shape functions, numerical integration of (B^T E B) from -1 to 1 gives the stiffness matrix K of the parallelogram plate bending element.

3.6 Numerical Method of Integration.

The integration is performed using a Gauss Quadrature technique. This procedure is a method of integrating polynomial functions by determining the values of the function at a certain number of sampling points (N),

multiplying those values by the appropriate weighting factors, and then summing the results.

For a given number of sampling points (N), an exact solution may be obtained for a polynomial of degree (2N-1) or less. The integration for the parallelogram element was performed using three point Gauss Quadrature on the normalized interval (-1, 1).

The sampling points and corresponding weighting factors are

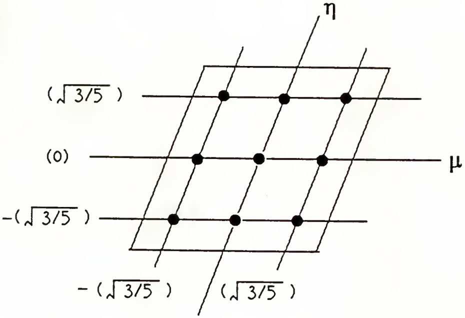
Location of Gauss points	Weighting factor
$-\sqrt{(3/5)} = -0.77459666$	5/9 = 0.555555555
0	8/9 = 0.888888888
$\sqrt{(3/5)} = 0.77459666$	5/9 = 0.555555555

Refer to Figure 3.4 for the Gauss points of the parallelogram element and refer to Appendix B for the stiffness matrix of parallelogram plate bending element.

3.7 Stress Matrix and Flexural Stresses.

After the elements have been assembled and the structure has been analyzed for nodal displacements, the generalized stresses at selected points in each element may be obtained from basic thin plate theory.

$$
\begin{aligned}
f_x &= E\,\epsilon_x = E/(1-PR^2)\ (\epsilon_x + PR\ \epsilon_y) \\
&= E/(1-PR^2)\ \{Z\ d^2w/dX^2 + Z\ d^2w/dy^2\ PR\ \}
\end{aligned}
$$



Location of Point	Weighting Factor
$-(\sqrt{3/5})$	0.55555 (5/9)
0	0.88888 (8/9)
$(\sqrt{3/5})$	0.55555 (5/9)

FIGURE 3.4 GAUSS INTEGRATION POINTS (N=3)
FOR PARALLELOGRAM ELEMENT

where f_x = stress in X - direction.

$\epsilon_{x,y}$ = strain in X,Y direction.

PR = Poisson's Ratio

Expressing the internal moment Mx in terms of fx

$$Mx = \int f_x \, Zdz = Et^3/12(1-PR^2) \, (d^2w/dx^2 + PR \, d^2w/dy^2)$$

Therefore, we can express the generalized internal moments as

$$\{ M \} = \{ Mxx, Myy, Mxy \} = D \{\phi\} = D [B] \{q\}$$

where [D] matrix = $\begin{bmatrix} Dx & D1 & 0 \\ D1 & Dy & 0 \\ 0 & 0 & Dxy \end{bmatrix}$ for an orthotropic material

$$= Et^3/12(1-PR^2) \begin{bmatrix} 1 & PR & 0 \\ PR & 1 & 0 \\ 0 & 0 & (1-PR)/2 \end{bmatrix}$$

Evaluating {M} at the i^{th} node,

$$\{ M \}i = \begin{bmatrix} Mx \\ My \\ Mxy \end{bmatrix}_i = D \; \{ \; (\epsilon) - (\epsilon_o) \; \}$$

where ϵ = [B] {q} (3.14)

$$= [D] \{ [B]\{q\} - (\epsilon_o) \} \qquad (3.15)$$

The product of the two matrices [D] and [B] is called the stress matrix, which relates nodal displacements at each degree of freedom to stress couples (internal moments). The results of numerical integration of the stress matrix are shown in Appendix C. Basic mechanics of material is used to obtain the flexural stresses from the stress matrix.

$$\{ ff \} = \begin{bmatrix} ffx \\ ffy \\ ffxy \end{bmatrix} = \frac{M\ C}{I} = 12 \frac{Z}{t^3} \cdot \begin{bmatrix} Mx \\ My \\ -Mxy \end{bmatrix}$$

where t = thickness of the plate.

Z = distance from the centroid of the plate.

3.8 Equivalent Nodal Loads.

3.8.1 Uniformly Distributed Loads.

When an uniformly distributed load of magnitude w acts over an element, the equivalent nodal loads corresponding to the nodal degrees of freedom at each element can be calculated as

$$F_D = A \int \{f\}^T w\ dA = w\ [J] \iint \{f\}^T\ dR\ dS$$

$$= w\ ab\ \mathrm{Sin}(\alpha) \iint f^T\ dR\ dS$$

where f^T is a transpose matrix of (1 x 12) shape function and A is a surface area of an element.

Numerical integration using three point Gauss Quadrature for each of the two axes of integration leads to

$$F_D = 4\ w\ ab\ \mathrm{Sin}(\alpha) \cdot \begin{bmatrix} 1/4 \\ -b/12 \\ a/12 \\ 1/4 \\ b/12 \\ a/12 \\ 1/4 \\ -b/12 \\ -a/12 \\ 1/4 \\ b/12 \\ -a/12 \end{bmatrix}$$

3.8.2 Thermal Gradients.

Assuming that the temperature varies linearly from the top to the bottom of the plate with a temperature differential of dT, the curvatures of the unrestrained plate element will be :

$$\epsilon o = \begin{bmatrix} -\,d^2w/dx^2 \\ -\,d^2w/dy^2 \\ 2d^2w/dxdy \end{bmatrix} = \begin{bmatrix} -\alpha\ dT/t \\ -\alpha\ dT/t \\ 0 \end{bmatrix}$$

where α = Coefficient of thermal expansion.

dT= Temperature differential (Top-Bottom).

t = Thickness of plate element.

Applying the principle of virtual work yields the equivalent nodal loads due to the uniform temperature variations, as follows.

$$F_T = \int B^T\ E\ \epsilon o\ dA$$

$$= ab\ Sin(\alpha) \iint B^T\ E\ \epsilon o\ dR\ dS$$

$$= ab\ Sin(\alpha) \iint B^T \begin{bmatrix} Dx & D1 & 0 \\ D1 & Dy & 0 \\ 0 & 0 & Dxy \end{bmatrix} (-\alpha\ dT/t) \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} dRdS$$

$$= E\ t^2\ (\alpha\ dT)/12(1-PR) \begin{bmatrix} -2\ Cot(\alpha) \\ a/Sin(\alpha) \\ -b/Sin(\alpha) \\ 2\ Cot(\alpha) \\ -a/Sin(\alpha) \\ -b/Sin(\alpha) \\ -2\ Cot(\alpha) \\ a/Sin(\alpha) \\ -b/Sin(\alpha) \\ 2\ Cot(\alpha) \\ -a/Sin(\alpha) \\ -b/Sin(\alpha) \end{bmatrix}$$

3.9 Local Moments and Stresses.

Calculated moments and stresses for each element are in the global X, Y direction. It is desirable, however, to determine moments and stresses in the inclined direction of the joint. A simple way of determining the local moments and stresses along the face of the skewed joint is to use Mohr's circle representation. We first treat the traction components τ_r and τ_{rs} on a plane whose normal makes angle A with the X axis. Refer to Figure 3.9.

3.9.1 Rotation of Second Order Stress Tensor.

The second order tensor in X and Y coordinates for plane stress [T_{xy}] is given by

$$T = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (3.21)$$

The components τ_{rs} are related to τ_{xy} by the tensor transformation equations.

$$\tau_{rs} = a_r^{\ x} a_s^{\ y} \tau_{xy}$$
$$\tau_{xy} = a_r^{\ x} a_s^{\ y} \tau_{rs}$$

or in matrix notation,

$$[T_{rs}] = [A]^T [T_{xy}] [A]$$
$$[T_{xy}] = [A] [T_{rs}] [A]^T \qquad (3.22)$$

where A is the matrix of direction cosines, $a_i^{\ j}$ = cos (Xi,Xj), of the angles between the Xi and Xj axes and A^T is its transpose.

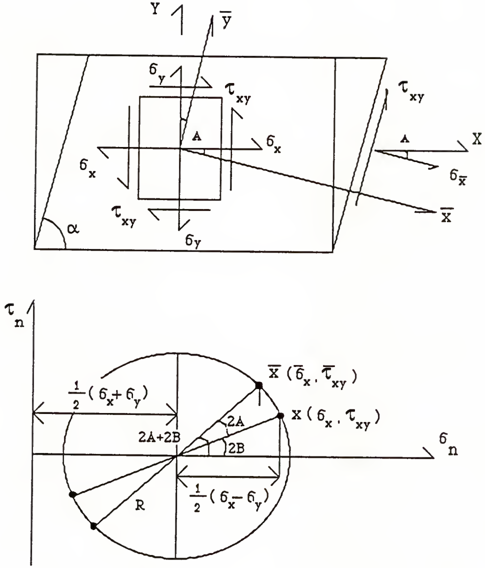


FIGURE 3.9 MOHR'S CIRCLE FOR PLANE STRESS

3.9.1.1 Direction Cosines of R and S Axes.

$$[A] = \begin{bmatrix} a_x^r = \cos(A) & a_x^s = \sin(A) & a_x^t = 0 \\ a_y^r = -\sin(A) & a_y^s = \cos(A) & a_y^t = 0 \\ a_z^r = 0 & a_z^s = 0 & a_z^t = 0 \end{bmatrix}_{3x3}$$

Transpose of A matrix,

$$[A]^T = \begin{bmatrix} \cos(A) & -\sin(A) & 0 \\ \sin(A) & \cos(A) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (3.23)$$

3.9.1.2 Rotation of Stress Tensor. ($Trs = A^T\ T\ A$)

First, premultiply $[A]^T\ [T]$.

$$\begin{bmatrix} c\ \sigma_x - s\ \tau_{xy} & c\ \tau_{xy} - s\ \sigma_y & 0 \\ s\ \sigma_x + c\ \tau_{xy} & s\ \tau_{xy} + c\ \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

then, postmultiply $([A]^T\ [T])\ [A]$

$$T_{rs} = \begin{bmatrix} \sigma_r & \tau_{rs} & 0 \\ \tau_{rs} & \sigma_s & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} c^2\ \sigma_x - cs\ \tau_{xy} - cs\ \tau_{xy} + s^2\ \sigma_y & cs\ \sigma_x - s^2\ \tau_{xy} + c^2\ \tau_{xy} - cs\ \sigma_y & 0 \\ cs\ \sigma_x + c^2\ \tau_{xy} - s^2\ \tau_{xy} - cs\ \sigma_y & s^2\ \sigma_x + cs\ \tau_{xy} + cs\ \tau_{xy} + c^2\ \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where c,s denotes cosine(A) and sine(A) respectively.

Therefore, the stress components at an arbitrary angle A are given by

$$\sigma_r = \sigma_x\ c^2 + \sigma_y\ s^2 - 2\ \tau_{xy}\ cs$$
$$\tau_{rs} = \tau_{xy}\ (c^2 - s^2) + (\sigma_x - \sigma_y) cs \qquad (3.24)$$

Using basic trigonometric formulae,

$$\cos^2 A = (1 + \cos 2A)/2 \qquad \sin^2 A = (1 - \cos 2A)/2$$

$$\cos^2 A - \sin^2 A = \cos 2A, \quad 2\sin A \cos A = \sin 2A$$

Therefore, $\sigma_r = 1/2\ (\sigma_x + \sigma_y) + 1/2(\sigma_x - \sigma_y)\cos 2A - \tau_{xy} \sin 2A$

$$\tau_{rs} = \tau_{xy} \cos 2A + 1/2\ (\sigma_x - \sigma_y) \sin 2A \qquad (3.25)$$

From Figure 3.9,

$$1/2\ (\sigma_x - \sigma_y) = R \cos 2B, \quad \tau_{xy} = R \sin 2B$$

where $R = \sqrt{[\ 1/4(\sigma_x - \sigma_y)^2 + \tau_{xy}{}^2\ ]}$,

$$\tan 2B = 2\ \tau_{xy}/(\ \sigma_x - \sigma_y\ ) \qquad (3.26)$$

Substituting (3.26) to (3.25) and trigonometric addition formulae,

$$\sigma_r = 1/2(\sigma_x + \sigma_y) + R(\cos 2B \cos 2A - \sin 2B \sin 2A)$$
$$= 1/2(\sigma_x + \sigma_y) + R \cos(2B + 2A)$$
$$\tau_{rs} = R(\sin 2B \cos 2A + \cos 2B \sin 2A)$$
$$= R \sin(2B + 2A)$$

3.9.2 Rotation of Stress Resultants. (Three Dimensional Stress Distribution)

In plate theory, stress resultants are expressed as distributions of force or couple per unit length along a section through the plate. Figure 3.10 shows an element of a plate, bounded by two parallel planes, with the z axis taken downward.

3.9.2.1 Stress Resultants.

Plate theory replaces the distributed normal stress σ_x per unit of area on the positive C-face of the element by the normal force Nx per unit length along AB, and bending moment

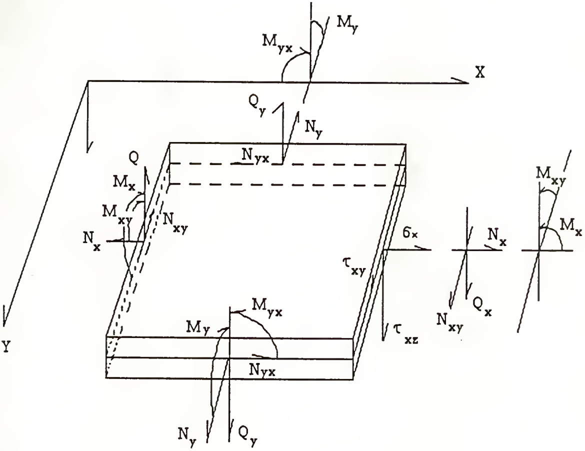


FIGURE 3.10 SIGN CONVENTION OF PLATE ELEMENT

Mx per unit length along AB,

$$Nx = \int_{-h/2}^{h/2} \sigma_x \, dz$$

$$Mx = \int_{-h/2}^{h/2} \sigma_x \, z \, dz$$

Nx and Mx are statically equivalent to the surface distribution of σ_x. The distribution of vertical shear stress τ_{xz} is statically equivalent to a vertical shear force Qx per unit length along AB.

$$Qx = \int_{-h/2}^{h/2} \tau_{xz} \, dz$$

The twisting moment Mxy per unit length due to horizontal shear forces is defined by

$$Mxy = -\int_{-h/2}^{h/2} \tau_{xy} \, z \, dz$$

$$Myx = \int_{-h/2}^{h/2} \tau_{xy} \, z \, dz$$

3.9.2.2 Rotation of Stress Resultants.

If we choose new R,S axes in the X,Y plane inclined at angle A to the old X-axis as shown in Figure 3.10, values of σr and τrs are again given by

$$\sigma_r = \sigma_x \, Cos^2A + \sigma_y \, Sin^2A - 2 \, \tau_{xy} \, CosA \, SinA$$

$$\tau_{rs} = \tau_{xy} \, (Cos^2A - Sin^2A) + (\sigma_x - \sigma_y) \, CosA \, SinA$$

If we multiply each of these equations through by z dz and integrate with respect to z from -h/2 to h/2,

$$\int \sigma_r \, zdz = \int (\sigma_x \, Cos^2A + \sigma_y \, Sin^2A - 2\tau_{xy} \, CosA \, SinA) \, zdz$$

$$\int \tau_{rs} \, zdz = \int [\tau_{xy}(Cos^2A - Sin^2A) + (\sigma_x - \sigma_y) CosA \, SinA] \, zdz$$

$$Mr = Mx \, Cos2A + My \, Sin2A + 2Mxy \, CosA \, SinA$$

$$-Mrs = -Mxy(Cos2A - Sin2A) + (Mx - My) CosA \, SinA$$

Using trigonometric formulae,

$$Mr = 1/2(Mx + My) + 1/2(Mx - My)Cos2A + Mxy\ Sin2A$$

$$Mrs = Mxy\ Cos2A - 1/2(Mx-My)\ Sin2A$$

Now let $R = \sqrt{[1/4(Mx-My)^2 + Mxy^2]}$

$$1/2(Mx-My) = R\ Cos2B$$

$$Mxy = R\ Sin2B,\ Tan2B = 2Mxy/(Mx-My)$$

Therefore,

$$Mr = 1/2(Mx+My) + R(Cos2B\ Cos2A - Sin2B\ Sin2A)$$

$$Mrs = R\ (Sin2B\ Cos2A + Cos2B\ Sin2A)$$

or,

$$Mr = 1/2(Mx+My) + R\ Cos(2B+2A)$$

$$Ms = 1/2(Mx+My) - R\ Cos(2B+2A)$$

$$Mrs = R\ Sin(2B + 2A)$$

CHAPTER 4
FINITE IN-PLANE STRESS ELEMENT

4.1 Introduction.

The purpose of this section is to analyze the in-plane effects of concrete shrinkage, subgrade friction, and thermal expansion or contraction on the pavement stress and joint displacements. This section presents a description of the finite element method used in the analysis including derivations of the system's stiffness matrix and load vectors.

4.2 Parallelogram In-Plane Stress Element.

The element used in this analysis is the four node, parallelogram, plane-stress, membrane element, which becomes a rectangular element when the skew angle equals 90 $^{\circ}$.

This parallelogram element has two degrees of freedom (DOF) at each node for a total of eight DOF per each element. Refer to the Figure 4.1. The eight DOF correspond to an X and Y axis displacements at each node, which are assumed to completely define the displaced shape of the element.

Let the generic displacements of the element be denoted by U = {u,v}, which is a 1x2 matrix and a function of x and y and let the nodal displacements (eight DOF) be noted by

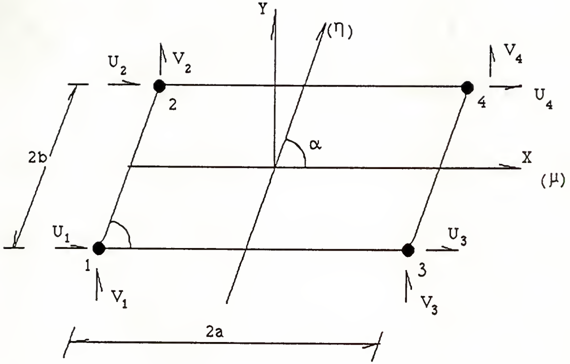


FIGURE 4.1 DEGREE OF FREEDOM FOR PARALLELOGRAM INPLANE ELEMENT

$q = \{u1, v1, u2, v2, u3, v3, u4, v4\}$.

4.2.1 Assumed Displacement Function.

Using a normalized coordinate system on the interval of -1 to 1 in the R and S axes, the displacement function in the element is assumed to be as follows.

$$u = c_1 + c_2 R + c_3 S + c_4 RS$$
$$v = c_5 + c_6 R + c_7 S + c_8 RS$$

4.2.2 Shape Function.

First, it is necessary to express the shape function U = (u, v) in terms of the vector of nodal displacements. We first consider a vector of eight unknown constants, {c}, such that

$$U = [g]\{c\}, \quad \text{where} \quad [g] = \begin{bmatrix} 1 & R & S & RS & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & R & S & RS \end{bmatrix}$$

{c} = unknown constants.

Evaluating the operator g at each node,

$$[h] = \{\ gi\ \} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Inverting the previous expression q = [h] {c} becomes

$$c = h^{-1} q, \quad \text{where } h^{-1} = 1/4 \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

Therefore, $U = [g]\{c\} = [g]\,[h]^{-1} q = f\, q$

where f is called as the shape function, which relates nodal displacements(q) to generic element displacement(U).

Shape function $f_i = g\ h^{-1} = \{1\ R\ S\ RS\}\ [h^{-1}]$

where, i = 1,2,3,4 node number.

$$f1 = 1/4\ (1 - R)(1 - S)$$

$$f2 = 1/4\ (1 - R)(1 + S)$$

$$f3 = 1/4\ (1 + R)(1 - S)$$

$$f4 = 1/4\ (1 + R)(1 + S)$$

We can simply check these shape functions using the Lagrange method with a positive unit displacement at the correct node while the other three nodes are fixed at zero displacement.

4.3 Strain - Displacement Relationships

Figure 4.1 shows an infinitesimal element that has been displaced by the amounts u and v in the x and y directions. Also shown on the figure are partial derivatives (or strains) that define incremental deformations of the element. The types of strains shown are

$$\epsilon = \{\epsilon x, \epsilon y, \epsilon xy\} = \begin{bmatrix} du / dx \\ dv / dy \\ du/dy + dv/dx \end{bmatrix}$$

The first two types, ϵx and ϵy, are normal strains in the x and y directions, and ϵxy is shearing strain. As before, they may be written in matrix form as

$$\epsilon = d\,u \quad \text{where} \quad d = \begin{bmatrix} d/dx & 0 \\ 0 & d/dy \\ d/dy & d/dx \end{bmatrix}$$

$$\text{and} \quad U = \{ u, v \}$$

Using the chain rule for partial derivatives,

$$d / dx = d/dR \; dR/dx + d/dS \; dS/dx = 1/a \; d/dR$$

$$d / dy = d/dR \; dR/dy + d/dS \; dS/dy = (-1/a)\mathrm{Cot}(\alpha) \; d/dR + (1/b)\mathrm{Cosec}(\alpha) d/dS$$

Applying this operator to matrix f (shape function) given in Section 4.2.2, we find that matrix B is

$$Bi = \begin{bmatrix} f_i,x & 0 \\ 0 & f_i,y \\ f_i,y & f_i,x \end{bmatrix} \quad (i = 1,2,3,4 \text{ node number})$$

The symbols f_i,x and f_i,y represent partial derivatives of f_i with respect to x and y. If we assume that the material is isotropic, stress-strain relationships can generally be written as

$$\sigma = [E] \{ \epsilon_T - \epsilon_0 \} \quad \text{where } \epsilon_T = \text{Total strain.}$$
$$\epsilon_0 = \text{Initial strain.}$$

$$\begin{bmatrix} \sigma x \\ \sigma y \\ \sigma xy \end{bmatrix} = \frac{E}{1 - PR^2} \begin{bmatrix} 1 & PR & 0 \\ PR & 1 & 0 \\ 0 & 0 & (1-PR)/2 \end{bmatrix} \begin{bmatrix} \epsilon x \\ \epsilon y \\ \epsilon xy \end{bmatrix}$$

4.4 Stiffness Matrix.

Having the necessary operators for stresses and strains, we can evaluate the element stiffness matrix.

$$K = \int B^T E B \, dV = t \, ab \, \sin(\alpha) \int_{-1}^{1} \int_{-1}^{1} B^T E B \, dR dS$$

Using two points Gauss Quadrature on the normalized interval of -1 to 1, the sampling points are

$$r1 = -1/\sqrt{3} \quad , \quad r2 = 1/\sqrt{3}$$

and the weighting factors are

$$w1 = 1.0 \quad , \quad w2 = 1.0$$

Results of integration appear in Table 4.1.

4.5 Equivalent Nodal Forces

It is necessary to define the nodal forces which are equivalent statically to the boundary stresses and distributed loads on the element due to the frictional forces, shrinkage loading, and temperature loadings. Each of the forces {Fi} must contain the same number of components as the corresponding nodal displacement {qi} and be ordered in the appropriate, corresponding directions.

4.5.1 Frictional Forces.

Since shrinkage and thermal effects involve no actual applied loads on a highway pavement system, stresses induced in the pavement will be those due to closing of the pavement joints and subgrade friction.

Table 4.1 STIFFNESS MATRIX OF PARALLELOGRAM IN-PLANE STRESS ELEMENT

4+R(T4+ +C2-C4)	P(-T3+ C3)+R(-T3+C3)	2+R(T2 -C2)	P(-T1- -C3)+R(-T1+C3)	-4+R(-T4 +C1)	P(T3+C3) +R(T3- C3)	-2+R(-T2 -C1+C4)	P(T1-C3)+R(T1- C3)
	T4+C2- C4+4R	P(-T1+ C3)+R(- T1-C3)	T2-C2+ 2R	P(T3-C3) +R(T3+ C3)	-T4+C1- 4R	P(T1-C3) +R(T1- C3)	-T2-C1 +C4-2R
		4+R(T4+ C2+C4)	P(-T3- C3)+R (-T3- C3)	-2+R(-T2 -C1-C4)	P(T1+C3) +R(T1+ C3)	-4+R(-T4 +C1) =(1,5)	P(T3-C3)+R(T3+ C3) =(2,5)
			T4+C2+ C4+4R	P(T1+C3) +R(T1+C3) =(3,6)	-T2-C1- C4-2R	P(T3+C3) +R(T3-C3) =(1,6)	-T4+C1- 4R =(2,6)
				4+R(T4+ C2+C4) =(3,3)	P(-T3-C3)+R(-T3- C3) =(3,4)	2+R(T2- C2) =(1,3)	P(-T1+C C3)+R(-T1-C3) =(2,3)
					T4+C2+C4 +4R =(4,4)	P(-T1-C3)+R(-T1+ C3) =(1,4)	T2-C2+ 2R =(2,4)
						4+R(T4+ C2-C4) =(1,1)	P(-T3+C C3)+R(-T3+C3) =(1,2)
							T4+C2- C4+4R =(2,2)

All terms multiplied by

$$D = \frac{E\ t\ b}{12(1-P^2)a} \operatorname{Sin}(\alpha)$$

P = Poisson's Ratio
R = (1-P)/2
T1= 2Cot(α)
T2= 2Cot(α)Cot(α)
T3= 4Cot(α)
T4= 4Cot(α)Cot(α)

C1= $2(a/b)^2$Cosec(α)Cosec(α)
C2= $4(a/b)^2$Cosec(α)Cosec(α)
C3= 3(a/b)Cosec(α)
C4= 6(a/b)Cot(α)Cosec(α)

[12 x 12 Symmetric Matrix]

The equivalent nodal loading due to friction can be derived using the differential work theory. The step in this process are as follows:

1) The generic X and Y displacements of a differential element {S}

$$\{ S \} = \begin{bmatrix} Sx \\ Sy \end{bmatrix} = \begin{bmatrix} U(R,S) \\ V(R,S) \end{bmatrix} = [f]\ \{q\} \qquad (4.1)$$

where [f] is a shape function of parallelogram plane stress element, as given in Section 4.2.2.
[q] is the vector of nodal displacement (8 DOF).
Refer to Figure 4.2 for details.

2) The vertical surface pressure on the differential element is given as

$$\{ \sigma_s \} = K_0 \cdot \begin{bmatrix} w \\ w \end{bmatrix} \qquad (4.2)$$

where K_0 = subgrade stiffness.
w = vertical deflection.

The planar friction pressure at any point beneath the pavement is given as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \end{bmatrix} = \phi \{ \sigma_s \} = K_0 \cdot \begin{bmatrix} w \\ w \end{bmatrix} \qquad (4.3)$$

where ϕ = coefficient of subgrade friction.

3) The vertical deflection w.

The vertical deflection of the area is given by

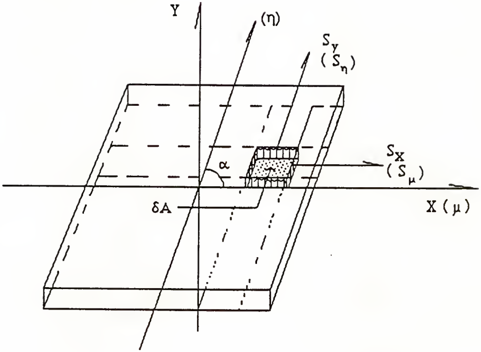


FIGURE 4.2 SURFACE PRESSURE DUE TO SUBGRADE STIFFNESS AND DISPLACEMENTS OF A DIFFERENTIAL ELEMENT

$$w = \{f_b\}\ \{q_b\} \quad (4.4)$$

where {fb} = 1x12 array of shape functions of plate bending element.

{qb} = 12x1 array of nodal displacements due to plate bending only.

The vertical deflections are obtained by running the FEACONS program using the same nodal mesh as that for the INPLANE program. The nodal displacements are stored in a data file which can then be used by the INPLANE program. Refer to the input manual of the FEACONS program to save the data.

4) The differential work done by the differential area is next expressed as

$$\begin{aligned} dw &= (\text{planar displacement}) \times (\text{planar friction}) \\ &= \{S\}^T\ \phi[\sigma_s]\ dA \\ &= \phi\ K_0\ \{q\}^T\ [f]^T\ [f_b]\ [q_b]\ dA \end{aligned}$$

Therefore, the total work done by the element is given as

$$W_T = \phi\ K_0\ \{q\}^T \int [f]^T\ [f_b]\ [q_b]\ dA \quad (4.5)$$

5) Next, the equivalent work for the element is formulated. For a set of equivalent nodal loads due to friction, the work performed by those loads going through a set of nodal displacements is

$$We = \{q\}^T\ [F_{fr}] \quad (4.6)$$

where $[F_{fr}]$ = equivalent nodal loads due to friction.

Equating $We = W_T$,

$$\{q\}^T [F_{fr}] = \phi K_0 \{q\}^T \int_{area} [f]^T [f_b] [q_b] dA$$

$$[F_{fr}] = \phi K_0 \int_{area} [f]^T [f_b] [q_b] dA$$

where $\int_A dA = \int_X \int_Y dX\, dY = |J| \int_R \int_S dR\, dS$

Therefore,

$$[F_{fr}] = \phi\, ab\, \mathrm{Sin}(\alpha)\, K_0 \int_R \int_S [f]^T [f_b]\, dRdS\, [q_b] \quad (4.7)$$

4.5.2 Shrinkage Loading.

The equivalent nodal loads due to uniform concrete shrinkage ϵ_{shr} are derived as follows.

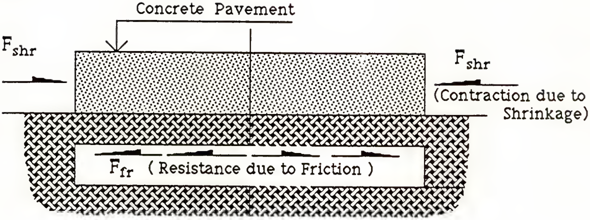
Setting an initial strain $\{ \epsilon_0 \}$ equal to ϵ_{shr},

$$[F_{shr}] = -t\, ab\, \mathrm{Sin}(A)\, \epsilon_{shr} \int_R \int_S [B]^T [C] \begin{bmatrix} 1 \\ 1 \end{bmatrix} dR\, dS \quad (4.8)$$

A careful check should be made to see if the magnitude of F_{shr} is greater than that of friction force. If so, the sum of F_{shr} and F_{fr} will be applied to the pavement. If not, the contraction force due to concrete shrinkage will be ignored. Refer to Figure 4.3 for details.

4.5.3 Temperature Loading.

The thermal gradient in the slab is assumed to be a second degree polynomial defined by three measured temperatures across the slab thickness, one at the surface (Tt) , one at the center (Tm), and the third at the bottom of the slab (Tb). These three temperature readings and a reference temperature (T_0) are then fitted to the polynomial



If $|F_{shr}| > |F_{fr}|$,

Then Total Active Load (PLOAD) :

$$PLOAD = F_{shr} + F_{fr}$$

Else $PLOAD = 0.0$

FIGURE 4.3 FRICTION-SHRINKAGE FORCE RELATIONSHIPS

equation $F(z) = A + Bz + Cz^2$. T_0 is the assumed temperature at which there are no thermal strains in the slab as shown in the Figure 4.4.

The equivalent nodal loading due to thermal effects can be computed By equating an initial strain (ϵ_0) equal to ϵ_{temp} in the previous equation as follows:

$$[\ Ft\ ] = \alpha \quad [B]^T\ [E] \begin{bmatrix} F(Z) \\ F(Z) \\ 0 \end{bmatrix} \quad dXdYdZ$$

The thermal gradient function F(Z) may be integrated over the slab thickness and the temperature readings substituted for the function constants to obtain

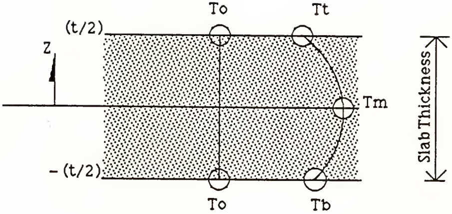
$$T = \int_{-t/2}^{t/2} F(z)\ dZ = t\ [Tm - T_0 + (Tt + Tb - 2Tm)/6\ ]$$

and for the normalized coordinate system,

$$[\ Ft\ ] = \alpha\ ab\ \mathrm{Sin}(\alpha) \int\int [B]^T\ [E] \begin{bmatrix} T \\ T \\ 0 \end{bmatrix} \quad dRdS$$

After computing equivalent nodal forces due to thermal gradients, compare the sum of shrinkage and thermal induced stresses with friction forces.

If an absolute magnitude of Fshr , |Fshr|, is less than |Ffr|, check to see if the sum of Ftemp and Fshr is greater than Ffr . If so, total applied load will be the sum of [Fshr], [Ftemp] and [Ffr]. Otherwise the total applied load will be zero. For more details, refer to Figure 4.5 and program INPLANE list.



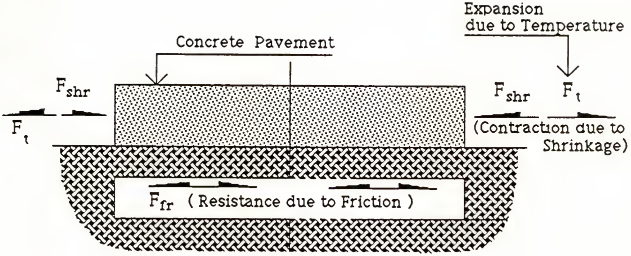
$$F(Z) = A + BZ + CZ^2$$

where $A = Tm$

$B = 1/t\,(Tb - Tt)$

$C = 2/t^2(Tt + Tb - 2Tm)$

FIGURE 4.4 ASSUMED DISTRIBUTION OF THERMAL GRADIENT ACROSS THE SLAB THICKNESS



If $|F_{shr}| > |F_{fr}|$,

Then Total Active Load (PLOAD)
Due to Thermal Gradients :
PLOAD = F_t

If $|F_{shr}| < |F_{fr}|$,

Then Let TV = $F_{shr} + F_t$

If $|TV| > |F_{fr}|$,

Then PLOAD = $F_t + F_{shr} + F_{fr}$

Else, PLOAD = 0.0

FIGURE 4.5 SHRINKAGE, THERMAL, AND FRICTION FORCES

4.6 Solution Process.

After the stiffness matrices of the structure have been assembled, shrinkage and friction force vectors are formed and the displacements are solved. Then, joint coordinates for displacements due to shrinkage and friction are adjusted. Next, apply full thermal loading to determine whether joints will close due to thermal effects.

As explained above, thermal force vectors are formed and are compared with friction forces, then the displacements are solved. The results are a set of simultaneous equations of the form,

$$\{ P \} = [K] \{ r \} - \{ p_0 \}$$

where [K] is an n x n stiffness matrix, {r} is an n x 1 array of unknown nodal displacements.

4.6.1 Equation Solver.

Using its properties of symmetry, [K] may be factored into two matrices such that

$$[L]\ [U] = [K] \tag{4.11}$$

where [L] and [U] are n x n, lower and upper triangular matrices,respectively. The matrix [U] may be further factored to obtain

$$[D]\ [L]^T = [U] \tag{4.12}$$

where [D] is an n x n diagonal matrix containing non-zero elements in its diagonal only. Substituting equation (4.12) to (4.11) results in

$$[K] = [L]\ [D]\ [L]^T \qquad \text{and,}$$

$$[L]\ [D]\ [L]^T \{r\} = \{\ P\ \} \tag{4.13}$$

The load vector {P} is then factored

$$\{P\} = [L]\ \{V\} \tag{4.14}$$

where {V} is solved, for using a forward substitution. Then back substitution is used to solve for the unknown displacements {r}. For details, refer to the Cholesky square root method (Ref. 2).

4.6.2 Element Stresses.

The next step is to calculate element stresses and strains from the nodal displacements. The most accurate estimation of the element strains are those at the Gauss points in the finite element. The coordinates of the Gauss points are inserted into the [B] matrix to get the total strain,

$$\{\ \epsilon\ \} = [B]\ \{q\}.$$

The initial strains due to shrinkage and temperature differentials are subtracted out and then stresses at the Gauss points are obtained. A least square fit method is used to get the linear interpolation equations for stresses on the four sets of stresses.

After the nodal stresses for each element are determined, and for those nodes that are common to more than one element, the average stresses are calculated.

CHAPTER 5
DEVELOPMENT OF ANALYTICAL TOOLS

5.1 Introduction

The research work in the evaluation and testing of rigid pavements in Florida produced a suitable analytical computer program called FEACONS. This program, originally developed by Kevin L. Toye and Shau Lei at the Department of Civil Engineering, University of Florida, was meant to perform a more thorough analysis of the bending behavior of a concrete pavement. Its major features are its ability to take into account both the pavement's load transfer capacity across its joints and the loss of contact between the slab and subgrade.

Despite the improvement that the FEACONS III program represents, another concern has arisen relative to skewed joints in the concrete pavement system. Subsequently, the present or fifth version of the program named FEACONS V was developed. The program considers the following factors in the analysis:

(1) The weight of concrete slabs.

(2) The subgrade voids beneath the concrete slabs.

(3) The effects of joints (load transfer at the joint).

(4) The effects of edges (effects of tie bars).

(5) The effects of temperature variations.

(6) The effects of skewed joints.

(7) The effects of dowel bars (both vertically and rotationally).

(8) The effects of nonlinear subgrade response analysis.

The program produces the following outputs:

(1) Initial deflections of pavement slabs due to self weight and thermal gradients.

(2) Deflections due to applied load.

(3) Final deflections are saved to run the INPLANE program.

(4) Moments, stresses and principal stresses in the slab.

(5) Moments and stresses along the face of a skewed joint expressed in terms of local coordinates, perpendicular and parallel to the direction of the joint.

(6) Maximum values of deflections, moments, stresses and principal stresses in the concrete slabs.

5.2 Modeling of Concrete Pavement System

The concrete pavement is modeled by using a three slab system with two intermediate joints. A concrete slab is modeled as an assemblage of finite plate elements with three degrees of freedom at each node as shown in Figure 5.1.

5.3 Load Transfer Mechanisms

5.3.1 Joints

Load transfer mechanisms across the joints between two

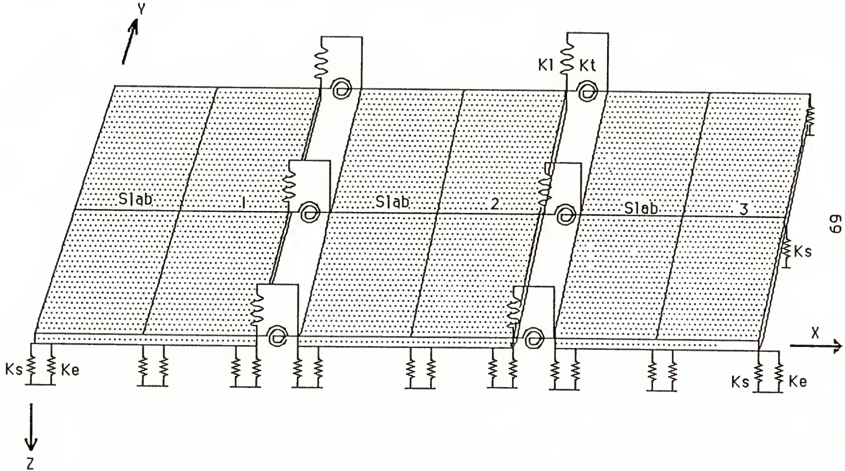


Fig. 5.1 Finite Element Modeling of a Three Slab Pavement System

adjoining slabs are modeled by shear (or linear) and rotational springs connecting the slabs at the nodes along the joint (Ref.15). Refer to Figure 5.2.

5.3.2 Dowel Bars

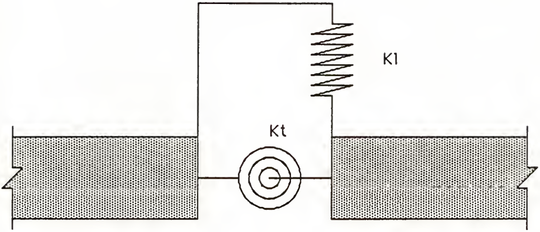
Looseness of the dowel bars is modeled by a specified slip distance, such that shear and moment stiffness become fully effective only when the slip distance is overcome. The effective dowel stiffness is modeled as varying linearly with the difference in deflection at the joint, when the difference in deflection is less than the slip distance. Refer to Figure 5.3 for details.

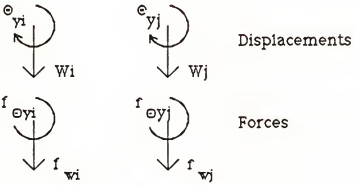
5.3.3 Edges.

Frictional effects of the tie bars are modeled by shear springs at the nodes along the edges.

5.3.4 Subgrade.

The subgrade is modeled as a Winkler foundation which consists of a series of vertical springs at the nodes. Subgrade voids are modeled as initial gaps between the slab and the springs at the nodes. A spring stiffness of zero is used when a gap exists. Either a linear or a nonlinear load deformation relationship for the springs can be specified. For the linear case, the subgrade stiffness remains constant as long as the slab and the subgrade are in contact with one another. For the nonlinear case, a load-deformation relationship of the following form is used:





$$\begin{bmatrix} f_{wi} \\ f_{\theta yi} \\ f_{wj} \\ f_{\theta yj} \end{bmatrix} = \begin{bmatrix} K_1 & 0 & -K_1 & 0 \\ 0 & K_t & 0 & -K_t \\ -K_1 & 0 & K_1 & 0 \\ 0 & -K_t & 0 & K_t \end{bmatrix} \times \begin{bmatrix} W_i \\ \Theta_{yi} \\ W_j \\ \Theta_{yj} \end{bmatrix}$$

Force -- Displacement Relationship

FIGURE 5.2 LINEAR AND ROTATIONAL SPRING ELEMENTS MODELING JOINT BEHAVIOR

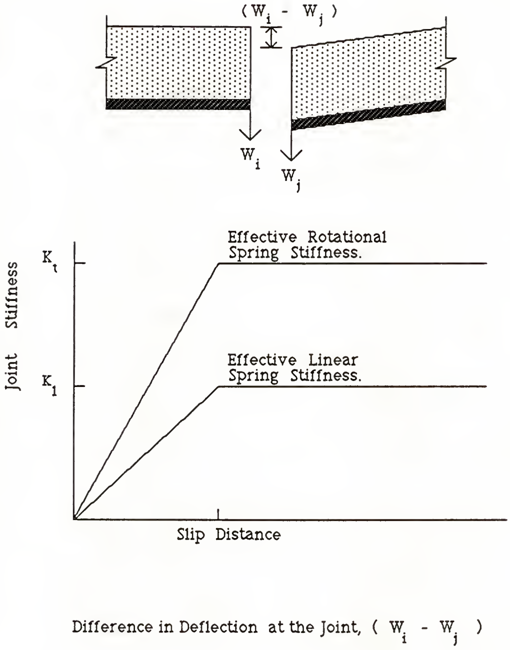


FIGURE 5.3 EFFECTIVE JOINT STIFFNESS AS FUNCTIONS OF THE DIFFERENCE IN DEFLECTION AT THE JOINT

$f = Aw + Bw^2$

where f = force/area

w = deflection

A and B = coefficients to be specified in the input. The subgrade stiffness is thus equal to $A + 2Bw$, which varies with the deflection.

5.4 Major Steps In The FEACONS V Program

The major computational steps are explained as follows: 1) The structural stiffness matrix is formed using slab dimensions and material properties. 2) The force vector due to weight of slabs, temperature variations and applied load is generated. 3) Deflections due to above loads are computed. 4) Internal moments and their maximum values in slabs are computed. 5) Flexural stresses and their maximum values are found. 6) Principal stresses including their direction and maximum values are computed. 7) The above procedure is repeated for the next load increment.
More details are described in the following sections. Refer to Figure 5.4 and 5.5 for the simplified flow chart of FEACONS V program.

5.4.1 Structural Stiffness Matrix

The explicit expression for the element stiffness matrix for the parallelogram plate bending element has been derived as shown in Chapter 3 and Appendix C. In the FEACONS program,

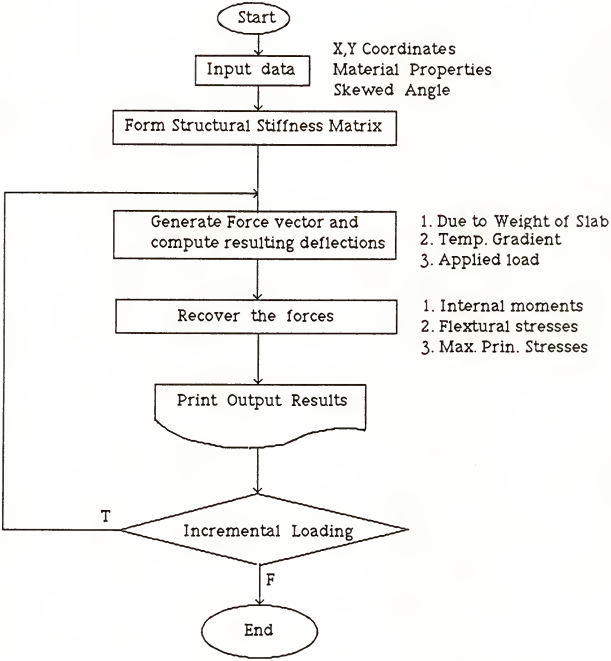


FIGURE 5.4 SIMPLIFIED FLOW CHART OF FEACONS V PROGRAM

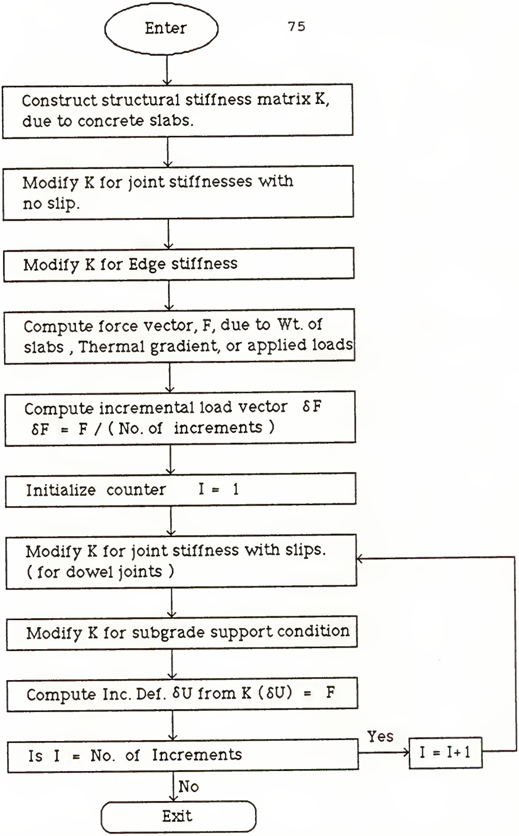


FIGURE 5.5 FLOW CHART FOR COMPUTING SLAB DEFLECTIONS

the nodal numbering is done automatically and starts in the lower left corner of a slab and continues in the Y-direction. The program then increments in the X-direction by rows. This method of numbering minimizes the bandwidth of the structural stiffness matrix and results in a half bandwidth equal to two times the number of nodes in the Y-direction plus four. The first step in this process is to define each element and form an index matrix that contains the structural node numbers corresponding to the element's local node numbers. Subroutine (KMAT) calculates the local stiffness matrix and subroutine (INSERT) is called to assemble the structural stiffness matrix.

5.4.2 Force Vector.

The force vector, F, due to the weight of the slabs, thermal gradients, or applied loads is first computed with the explicit expressions derived in Chapter 3. Subroutines (SLABWT), (TEMPER), and (FORCEU) assemble the force vectors caused by the slab weight, temperature variations and applied loads, respectively, as with the formation of the stiffness matrix.

5.4.3 Slab Deflections.

The force vector is first divided by the number of specified load increments to obtain the incremental load vector, dF. The deflections dU, caused by the incremental force vector are computed from the stiffness equation

$$K\,(dU) = dF$$

where K= structural stiffness matrix

dU= vector of incremental nodal deflections

dF= vector of incremental nodal forces.

After each computation of incremental deflections, the structural stiffness matrix K is modified according to the new deflected positions of the slab. The new K is then used to compute the incremental deflections for the next load steps. However, if structural stiffness does not change throughout a computational step, only an increment of one needs to be satisfied for that computational step. This applies to the case when the subgrade contact conditions do not change throughout a computational step and no slip is specified at the joints.

5.4.4. Internal Moments and Stresses.

The internal moments per unit length at the nodes is calculated from the final nodal deflections. Subroutine (STRMAT) calculates the 12x12 elemental stress matrix (SMAT) with the explicit expressions derived in Chapter 3 by extracting the 12x1 elemental displacement vertor(EDELTA). Therefore, the elemental stress couple vector (Moment) can be found by the multiplication of (SMAT) and (EDELTA). The moment intensities at each node are the two bending moment intensities Mx and My, and the twisting moment intensities Mxy. Mx, My, and Mxy are the bending moment intensities

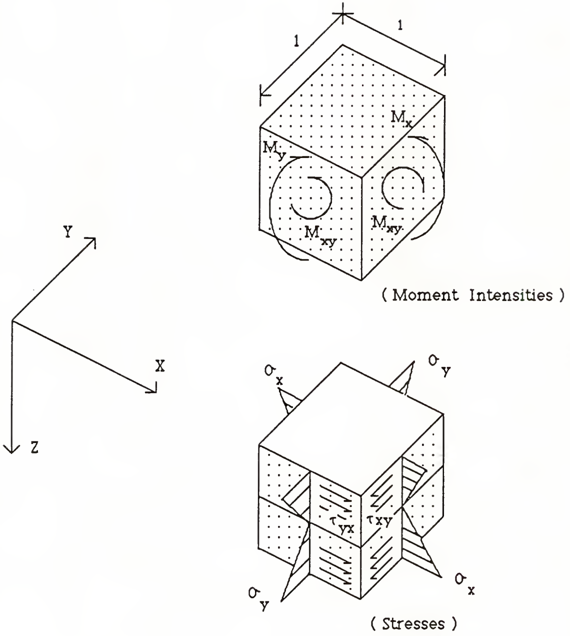


FIGURE 5.6 MOMENT INTENSITIES AND STRESSES AT A NODE

due to fx, fy, flexural stress in the x and y direction and due to fxy, shearing stress in the xy direction respectively. The direction of these moment intensities and the corresponding stresses are shown in Figure 5.6. Subsequently, flexural and shearing stresses are calculated from the moment intensities using the following equations from classical thin plate theory.

$$fx = \frac{12\ z}{t^3}\ Mx$$

$$fy = \frac{12\ z}{t^3}\ My$$

$$fxy = \frac{12\ z}{t^3}\ Mxy$$

where t= thickness of the slab.

z= distance from the centroid of the slab.

The program computes the stresses at the bottom of the slab by setting z=t/2 in the equations.

5.5 Development of INPLANE II program

Despite the many features of FEACONS, the planar effects of concrete shrinkage and thermal effect were still ignored. The computer program, INPLANE (Ref.16), originally developed by Michael F. Landry as his master's thesis at the Department of Civil Engineering, University of Florida, analyzes the in-plane effects of a concrete pavement systems subjected to

concrete shrinkage, subgrade friction, and thermal expansion or contraction, such that when INPLANE is used with the FEACONS program, a concrete pavement model, a previously unattained degree of rationality is included in the structural analysis.

In this study, the initial program developed by Landry was expanded and refined in the following areas:

1) The rectangular plate element of the concrete pavements was expanded to a parallelogram element to consider the effects of skewed joints on the concrete pavement .
2) The indexing of the DOF for the INPLANE program was modified to correspond to that of FEACONS program.
3) Relationships among the thermal forces, concrete shrinkage forces, and frictional forces were refined to determine the fully active forces beneath the slabs.

5.6 Computer Modeling and Boundary Conditions.

The concrete pavement is modeled in a manner similar to the FEACONS program by using a three slab system with two intermediate joints. To satisfy structural stability and equilibrium conditions, we must restrain some nodal points on the slab.

Because of symmetricity, the nodes along the slab's centerlines will have zero displacements perpendicular to those centerlines. Roller supports were placed along the

lateral (Y-direction) centerlines of the slabs, and pins at the centers. These boundary conditions will satisfy structural stability by preventing the nodes along the lateral centerlines from displacing perpendicular to those centerlines. In addition, Figure 5.7 shows which directions the friction forces act in response to the concrete shrinkage and thermal gradient effects. The directions of the friction forces at a node depend on which quadrant that node is in. The directions of the friction forces shown in Figure 5.7 are reversed for thermal expansion.

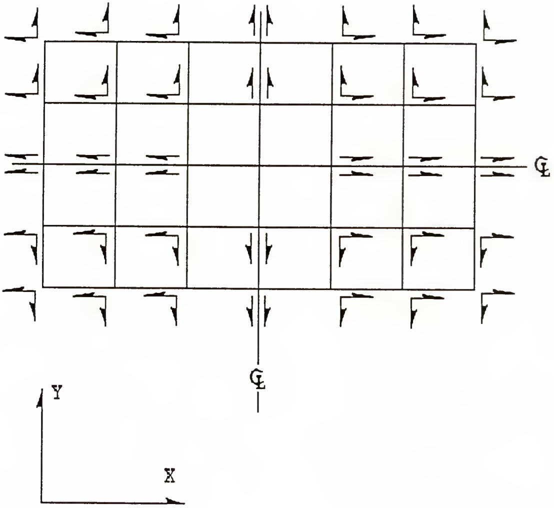


FIGURE 5.7 DIRECTIONS OF NODAL FRICTION FORCES
ACTING IN RESPONSE TO COMPRESSIVE LOADING

CHAPTER 6
CONCRETE PAVEMENT PARAMETERS

6.1 Introduction

It is very important to recognize that pavement systems are exposed to many environmental factors which could cause the distress and failure of pavements without the application of wheel load. As Sargious (Ref.17) has noted, the general mechanisms by which the environment influences pavement behavior and performance are (1) the effect on engineering properties of component materials, such as physical strength and tractive resistance. (2) the effect on the integrity of materials, such as durability and physicochemical disintegration. (3) the effect on volumetric change and the resulting internal stress equilibrium in the pavement system. In classic pavement design, these mechanisms have been closely related to climatic factors, i.e., temperature and moisture, in performance analyses. Many researchers have characterized the moisture and temperature in the pavement system as functions of space and time.

In addition to these, there are several other parameters which have caused considerable concern: (1) effects of wheel load magnitude and location, (2) effects of various joint, edge and subgrade conditions. In this dissertation, major

focus will be given to the study of the effect of each parameter on the basic equilibrium and stress conditions of a pavement system.

6.2 Wheel Load for Highways.

The great variety of traffic using highways makes it necessary to replace the traffic loads by a simple loading system that can be easily used in pavement design. Although passenger cars form the highest percentage of vehicles using the highway system, their loads are small compared with those of trucks. The loads produced by the movement of heavy trucks are critical to pavements and so will be discussed in some detail. According to Sargious (Ref.17), the live highway loads that generally affect pavement design consist of a combination of single-unit and multiple-unit vehicles. Single units include all 2-, 3- and 4- axle-unit trucks and all buses. The axles are either single or tandem, and are provided with single or dual tires. Multiple units include 3-, 4- and 5-axle tractor trucks with semi-trailers, and all full trailer combinations. The overall length of single powered commercial vehicles, in North America, varies between 35 ft (10.7 m) and 40 ft (12.2 m) and the length of combinations, consisting of a tractor, semi-trailer and/or full trailer varies between 65 ft (19.8 m) and 70 ft (21.4 m). Maximum width is about 8.5 ft (2.6 m) and maximum height is between 13.5 ft (4.1 m) and 14.5 ft (4.4 m).

It should be noted that truck-axle spacing and weight limits are variable quantities in different countries. Even more than that, these values vary from one state to another in North America. The maximum allowable load for single-unit trucks, for example, is between 28 kips (12.6t) and 60 kips(27.3t), according to the number of axles and wheels, and according to the specifications for pavement design in each state. Similarly, for a tractor with a semi-trailer and/or a full trailer, the maximum allowable load varies between 60 kips (27.3t) and 135 kips (61 t). The maximum single-axle load ranges between 16000 lb (7.1 t) and 22400 lb (10 t). The maximum tandem-axle load ranges between 28000 lb (12.5 t) and 40000 lb (918 t). Tandem spacings range between 40 in (100 cm) and 53 in (135 cm). Spacing centerline to centerline between dual wheels is about 13.5 in (34 cm) and tire pressures are 60-90 psi (4.2-6.3 kgf/cm2).

In many design procedures used in North America, a standard load is chosen for the design. The standard load commonly used is the 18000 lb (8100 kg) single-axle load with dual wheels on each side. Other standard loads are sometimes used, for instance, the 22400 lb (10100 kg) single-axle load, 32000 lb (14500 kg) tandem-axle load or 40000 lb (18100 kg) tandem-axle load.

Extensive studies on both flexible and rigid pavements have provided factors for converting various axle loads to an equivalent number of applications of the 18000 lb single-

axle load. These equivalency factors, for any given single- or tandem-axle load, express the number of 18000 lb single-axle load applications that will produce the same amount of pavement deterioration as that produced by one application of the given axle load. The load equivalency factors for flexible pavement are different from those for rigid pavements.

The trend towards heavier trucks, equipped with multiple wheel assemblies, has emphasized the need in pavement design for a simple standard, irrespective of arrangement or configuration of the wheels. In many cases, the conversion of multiple-wheel loads to equivalent single wheel loads offers the designers of flexible and rigid pavements a convenient means for both the design and evaluation of highway and airfield pavements. The interrelation of loading effects produced by single- and multiple wheel assemblies has been determined through theoretical analysis, laboratory investigations, accelerated traffic tests on prototype pavements and investigations of pavement performance. It consists of determining the magnitude and contact area of a single-wheel load which, when placed centrally on a concrete slab, will produce the same maximum stress as that produced by two or four closely spaced loads. These loads may represent the axle load of a truck or the dual-tandem load of an aircraft undercarriage.

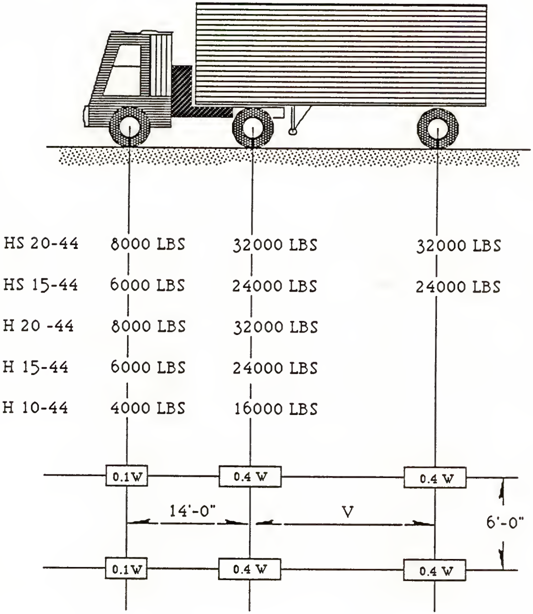
The AASHTO specification shows two systems of highway

loading, the H loadings and the HS loadings. The H loadings consist of a two-axle truck and the HS loadings consist of a tractor truck with semi-trailer as shown in Figure 6.1.

In this parametric study, the H-20 and the HS-20 AASHTO loadings are considered as a maximum wheel load (48 kips) with a safety factor of 1.5. We also calculate the load-induced stresses and deflections for intermediate loads of 16, 24, 32, 40, and 48 kips. These calculations facilitate identification of the nonlinearity of the pavement system response.

6.3 Temperature Variations.

The temperature variation through a pavement slab is not linear. Changes in ambient temperature during a 24-hour thermal cycle induce a wide variety of temperature gradients in the slab. Since methods for predicting temperatures in pavement systems are complex, no simple method can be effectively used to describe the nature of temperature. Armaghani (Ref.8) performed two major tests to study temperature response of concrete pavement. The first test involved measuring temperatures of the test road slabs. The second test included measurement of temperature-induced slab displacements in the vertical and horizontal directions. His research shows that minimum temperature at the surface is reached between 6:00 a.m. and 8:00 a.m. While maximum temperature, at the same position, is reached between 1:00 p.m. and 3:00 p.m. The temperature differential between top



W = Combined weight on the first two axles which is the same as for the corresponding H truck
V = Variable spacing - 14 ft to 30 ft inclusive

FIGURE 6.1 AASHTO H- AND HS- LOADINGS

and bottom of slab is responsible for the magnitude and direction of slab curling. As observed, the temperature differential is negative at night and positive during daytime hours. It is very important to note that maximum negative temperature differential occurs at approximately 6:00 a.m., and maximum positive temperature differential occurs at about 1:00 p.m.

6.4 Subgrade and Joint Stiffnesses

As discussed in Section 2.2, load is transferred across joints through shear and moment resistance by two means, interlocking of aggregate and dowel bars. The degree of load transfer is dependent on the joint stiffness, which is governed by shear and moment resistance. In return, the joint stiffness affects the load response of the pavement. Higher joint stiffness causes greater load transfer, which generates low deflections and lower overall stresses in pavement slabs, even stresses adjacent to joints are higher. It might be noted that the joint stiffness varies sharply with temperature, subgrade conditions, joint details and other factors. As such it can be difficult to define quantitatively. The stiffness factors can be found for a particular pavement under particular conditions by correlating measured linear pavement response, from Falling Weight Deflectometer test data.

For the parametric study, six joint stiffness

combinations, as listed in Table 6.1, are selected for use in calculating pavement response. Three different subgrade conditions, soft, medium and hard spring stiffness are also used, for a total of 18 combinations, as shown in Table 6.1.

Table 6.1 COMBINATIONS OF JOINT AND SUBGRADE STIFFNESS

Subgrade stiffness	joint stiffness Kr(k)	Kl(ksi)
soft (0.1 kci)	low (1000)	10 200 750
	high (10000)	200 750 1500
medium (0.3 kci)	low (1000)	10 200 750
	high (10000)	200 750 1500
Hard (1.2 kci)	low (1000)	10 200 750
	high (10000)	200 750 1500

CHAPTER 7
PARAMETRIC STUDIES OF CONCRETE PAVEMENT
(Computer Analysis)

7.1 Introduction

Parametric studies were performed to investigate the effects of various important pavement parameters on the structural response of concrete pavements. The parameters considered were those which were relevant to the I-75 concrete pavement in Sarasota and Manatee counties, which show extreme pavement failure and distress. These concrete pavements were constructed on an unbonded econocrete subbase with tied and dowelled joints, variable slab lengths, and skewed joints. These input parameters are explained in the following sections.

7.2 Input Parameters

As discussed in Chapter 6, there are several factors which could cause the distress and failure of concrete pavements. The factors to be considered in this study include subgrade stiffness, joint stiffness, temperature variations, and joint skew.

7.2.1 Features of the I-75

The general dimensions of the I-75 are given as follows:

1) Slab lengths (variable spacings): 16, 17, 22 and 23 ft.

2) Slab width: 12 ft

3) Slab thickness: 9 and 12 inch

4) Skew angle: 9.4623 ^{o} in degrees

5) Concrete-econocrete interface: Unbonded

6) Number of lanes: 6 lanes

7) Shoulder: Econocrete-tied

8) Pavement opened to traffic: 1980 ~ 1982

7.2.2 Choice of Parameters for the Analytical Study

Subgrade stiffnesses were measured using the FWD as described by Armaghani (Ref.8), and ranged as follows:

i) Subgrade spring stiffness, Ks

a) Ks = 0.25 kci

when dT = -30 ^{o}F during winter testing.

b) Ks = 0.35 kci

when dT = -50 ^{o}F during the summer.

c) Assume 0.25 kci-0.35 kci as the range of Ks values of the test roads.

Following values used for soft subgrade: 0.1 kci

medium subgrade: 0.3 kci

hard subgrade: 1.2 kci

(Econocrete subbase)

ii) Material Properties of concrete

i) Density of concrete: 140 pcf

ii) Elastic modulus of concrete: 5290 ksi

iii) Poisson's ratio of concrete: 0.2

iii) Joint stiffness: Kr, Kl

Rotational stiffness Kr(k-in/in)	Linear stiffness Kl(ksi)
1000 (low)	10 200 750
10,000 (High)	200 750 1500

3) Spring coefficient for the edges,Ke

Frictional effects of tie bars on the slab edge.

Ke = 10 ksi when Ks = 0.25 kci test road.

4) Factors of temperature differential effect

i) Coefficient of thermal expansion = 0.000006 ($1/^{o}F$)

ii) Temperature differentials between the top and the bottom surface were measured.

7.2.3 Wheel Load Arrangements

Both single and tandem axle loadings were used in the study. Each of these were placed both centered on the pavement and adjacent to the edge of the pavement, both in the vicinity of the joint and midslab.

1) Magnitudes of single axle load

The standard AASHTO loadings are placed with an overload factor of 1.25 and 1.5.

AASHTO	H10	16 kips	per axle
AASHTO	HS15 (H15)	24 kips	per axle
AASHTO	HS20 (H20)	32 kips	per axle
max.	possible load	32x1.25=40kips	per axle

max. ultimate load 32x1.5 =48kips per axle

2) Wheel load arrangements

Types of truck wheel arragements can be divided into two basic categories, including (1) single and dual wheels, (2) single and tandem axles (Ref.18). A wheel load on one set of dual tires will be one half the axle load as shown in Figure 7.1. For the skewed joint pavement, Figure 7.2 illustrates that how these axle loads were placed on the finite parallelogram meshes. Figure 7.3 shows four different equivalent single axle load positions and tandem axle positions.

7.3 Input Meshes

As discussed in Chapter 5, a concrete pavement is modeled by using a three slab system with two intermediate joints. As we divided a slab into hundreds of finite elements, two different input meshes are considered. One is for the joint loading case, in which an axle load is placed on the joint as shown in Figure 7.4. The other is for the mid-slab loading case, which an axle load is placed on the transverse mid-slab as shown in Figure 7.5.

7.4 Effects of Skewed Joint

The maximum stress in the slab caused by a 40 kip single axle load (which is two 20 kip wheel load at 6 feet apart) applied at a skewed joint was computed and compared to

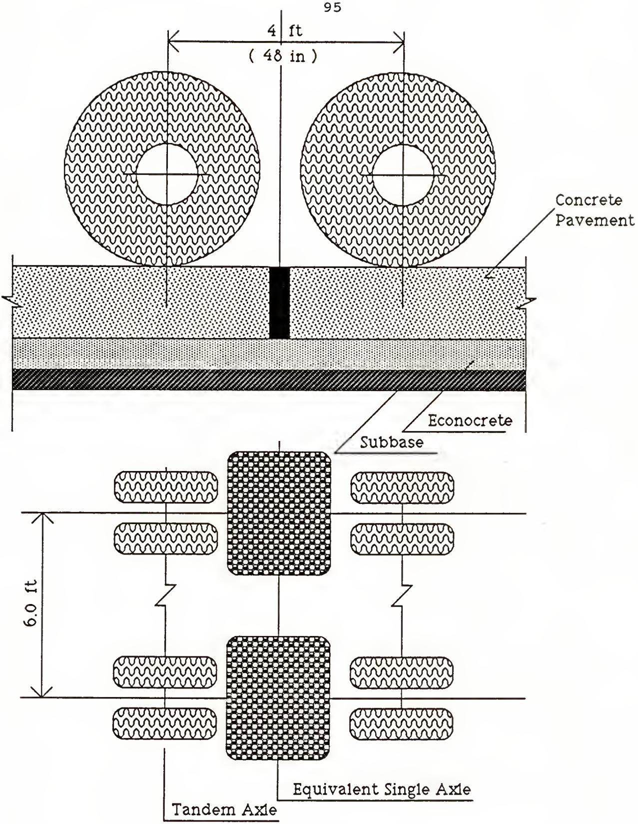


FIGURE 7.1 PLAN OF AXLE LOAD

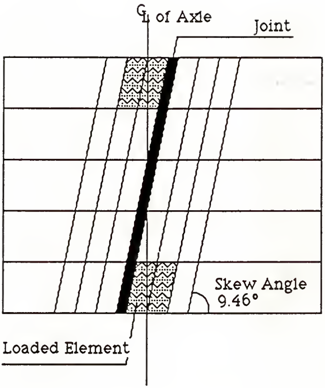


FIGURE 7.2 UNIFORM AXLE LOAD ON SKEWED JOINT ELEMENT

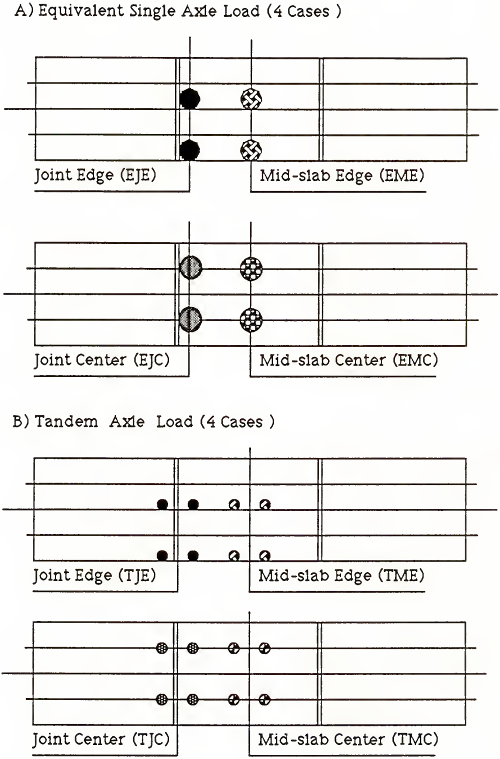


FIGURE 7.3 LIVE LOAD LOCATIONS FOR PARAMETRIC STUDIES

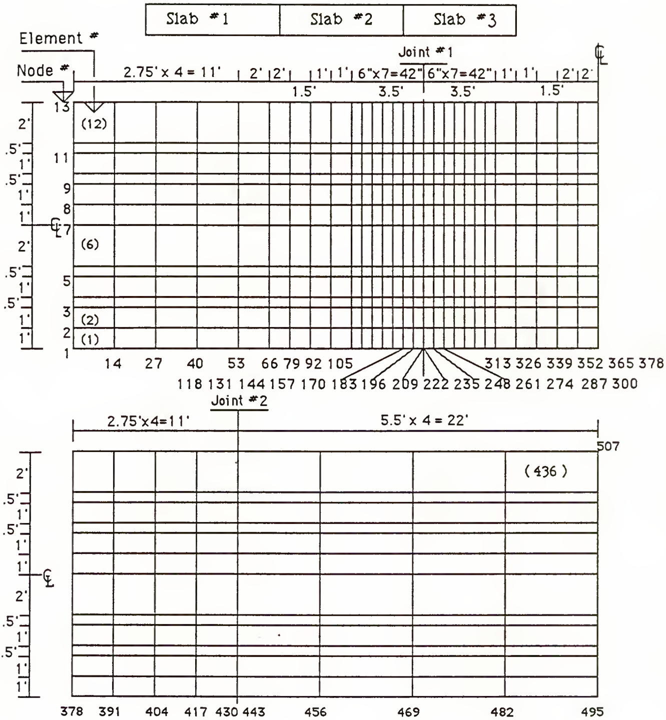


FIGURE 7.4 INPUT MESHES FOR JOINT LOADING

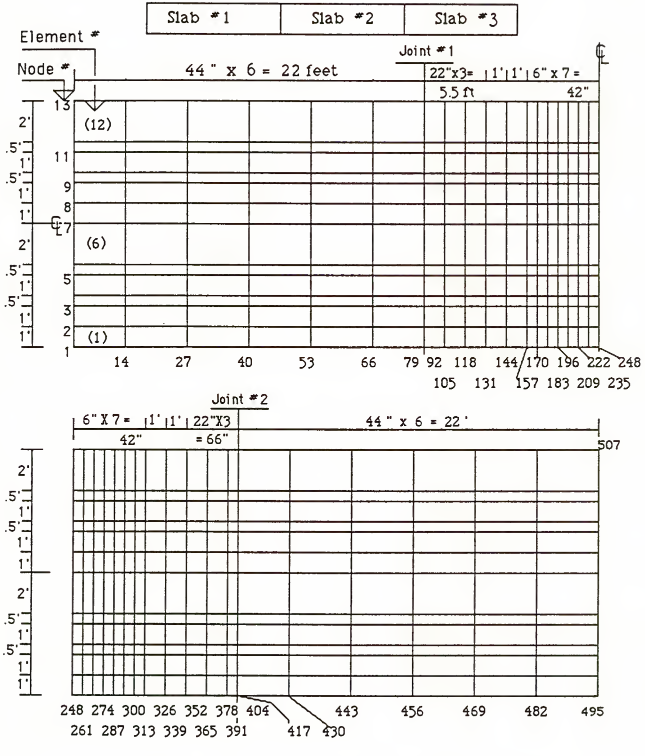


FIGURE 7.5 INPUT MESHES FOR MID-SLAB LOADING

that caused by the same 40 kip axle load at a regular transverse joint.

7.4.1 Joint Edge Loading

First, the effects of skewed joints on the structural response of a concrete pavements due to the joint-edge load were analyzed using the FEACONS V program with the following input parameters:

1) Slab length = 22 feet, and width = 12 feet
2) Slab thickness = 9 inches
3) Subgrade modulus Ks = 0.175 kci
4) Edge stiffness Ke = 10 ksi
5) Joint stiffness Kl = 10 Ksi, Kr = 16000 Kips

The maximum flexural stress caused by a 40 kip joint edge load and a +20 $^{\circ}$F temperature differential was computed with skewed angles A = 0, 9.4, 18.4 $^{\circ}$ and are tabulated in Table 7.1.

Table 7.1 EFFECTS OF SKEWED JOINTS ON MAXIMUM STRESSES DUE TO A JOINT EDGE 40 KIPS AXLE LOADING

Input data : Ks = 0.175 ksi, dT= +20 $^{\circ}$F
Ke = 10 ksi, Kl=10 ksi, Kr = 16,000 k

Skew	maximum stress (psi)			maximum principal stress(psi)		
Angle	σxx	σyy	σxy	σ1	σ2	Tmax
A=0	598.3 (242N)	678.3 (229N)	93.4 (270N)	678.9 (229N)	594.7 (242N)	250.5 (209N)
A=9.46	607.5 (203N)	743.1 (216N)	105.4 (507F)	754.2 (216N)	594.9 (203N)	293.8 (222N)
A=18.4	665.1 (177N)	628.2 (177N)	127.0 (507F)	684.8 (177N)	608.4 (177N)	438.1 (222N)

* The numbers in parentheses denote node numbers at which maximum stress occurs. N denotes near the loading zone and F denotes far away from the loading zone.

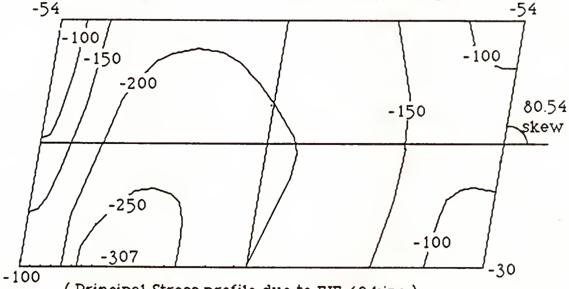
The maximum computed bending and shear stress at the regular transverse joint (A=90 ^{o}) with dT = +20 ^{o}F temperature differential were a longitudinal tensile stress of σx = 598 psi at the top surface of the slab and just to the right of the joint (Node number 242), and a transverse tensile stress of σy = 678 psi, located near the center line along the joint, and a shear stress of σxy = 93 psi, located very near the loading point.

A similar analysis was done with a skew angle of 9.46^{o}, but using the same slab length and input parameters. The maximum computed bending and shear stresses were a longitudinal tensile stress of σx = 608 psi, located near the joint, a transverse tensile stress of σy = 743 psi and a shear stress of σxy = 105 psi.

Referring to Table 7.1, which gives results for joint edge loading, the major principal stress, $\sigma 1$ increases and then decrease with increased skew angle. The minor principal stress increases, but only for the highest skew angle, while the shear stresses increases continually. In no case, however, are the variations in stress truly striking.

Another way of evaluating the effect of skewed joints on concrete pavement is through a maximum principal stress profile, showing lines of constant principal stress on the surface of the slab. This provides a very concise way of looking at the distribution of stresses, and will be used throughout this chapter. Figure 7.7 shows the effects of

Input Data : Slab Dimension Length=22 ft
Width = 12ft Thick = 9 in Skew = 9.46°
Material Property Ec = 5290 PR = 0.2
Stiffness Ks = 0.1 Kr=1000 , Kl=10 Ke=10
dT = -10 °F Wheel Load = EJE 40 kips



(Principal Stress profile due to EJE 40 kips)
(- sign denotes compression at the bottom in psi)

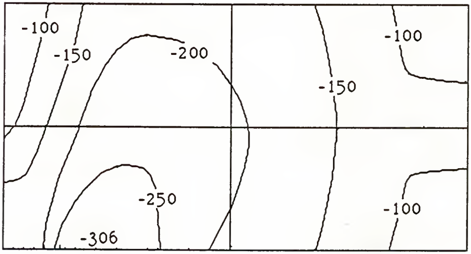


FIGURE 7.7 EFFECTS OF SKEWED JOINT ON PRINCIPAL STRESS PROFILE

skewed joints on the maximum principal stress profile associated with an equivalent joint edge load of 40 kips and a -10^{o}F temperature differential. It is seen that even though the variations in maximum stress displayed in Table 7.1 are not drastic, these higher stress levels are reached closer to the corner when the joint is skewed. This may well have a more powerful effect on behavior, even though there are small changes in maximum stress.

7.4.2 Mid-Slab Edge Loading

Second, the maximum stresses in the slab caused by the combination of a 40 kip single axle load at the mid slab edge and a temperature differential of +20 ^{o}F in the slab were computed for both skewed and perpendicular joint and the results are shown in Table 7.2.

Table 7.2 EFFECTS OF SKEWED JOINTS ON MAXIMUM STRESSES DUE TO A MID-SLAB LOADING OF 40 KIPS

Input data : Ks = 0.175 ksi, dT= +20 ^{o}F
Ke = 10 ksi, Kl=10 ksi, Kr = 16000 k

Skew Angle	maximam stress (psi) σxx	σyy	σxy	maximum principal stress(psi) $\sigma 1$	$\sigma 2$	Tmax
A=0	744.3 (249N)	564.9 (255N)	77.6 (507F)	744.3 (249N)	564.9 (255N)	337.6 (248N)
A=9.46	747.8 (275N)	575.7 (255N)	101.7 (507F)	748.0 (275N)	575.4 (255N)	335.9 (274N)
A=18.4	759.6 (275N)	603.2 (229N)	120.4 (505F)	760.3 (275N)	601.7 (229N)	353.9 (287N)

* The numbers in parentheses denote node numbers at which maximum stress occurs. N denotes near the loading zone and F denotes far away from the loading zone.

Table 7.2 shows that with an equivalent mid-slab edge load of 40 kips and + 20 ^{o}F temperature differential, the maximum flexural and principal stresses are slightly increasing as the skew angle grows. Therefore, it can be concluded that the use of a skewed joints in concrete pavements induces slightly higher stresses, which should not in themselves affect structural performance. However, the higher stress are attained closer to the corner, which may or may not affect structural performance significantly.

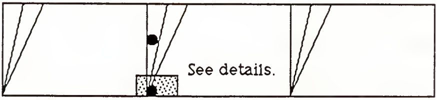
7.4.3 Stress Investigations For The Corner Crack Problems With Skewed Joint

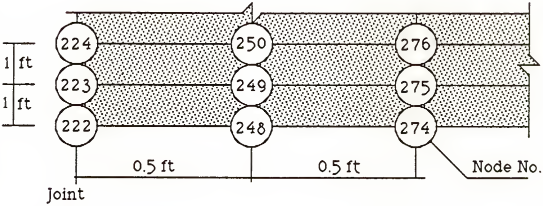
One aspect of the deterioration of the I-75 skewed joint concrete pavement is corner cracking, which occurs mostly at the acute corner of a slab. It was decided to supplement the conclusions of the stress levels very close to the corner. The results of these analyses are shown in Table 7.3.

As expected, Table 7.3 shows that flexural stress in the X- direction due to the 40 kip joint edge load decreases very slightly as skew angle grows. However, the flexural stress in the Y - direction increases as skew angle increases significantly for the higher skew angles. This result, coupled with the presence of higher principal stresses closer to the joint, may contribute to the higher frequency of corner cracking on the skewed joint pavement.

TABLE 7.3 STRESS INVESTIGATION AT THE JOINT EDGE CORNER

Input Parameter : Slab L=22 ft, W=12 ft, t=9 in
Concrete Ec=5290, PR=0.2, Joint Edge Load=40 kip
Stiffness Ks=0.175, Ke=10, Kl=10, Kr=16000
Zero Temperature Differential case.





	Skew Angle	Node No.								
		222	223	224	248	249	250	274	275	276
σ_x psi	0	249	249	203	207	210	178	-16	10	36
	9.46	220	246	201	237	210	151	8	13	18
	18.46	160	241	192	235	206	120	11	11	-0.2
σ_y psi	0	47	224	92	34	160	77	1	73	45
	9.46	-52	253	186	-2.2	188	128	-33	95	74
	18.46	-316	227	217	-118	200	142	-144	110	79

7.5 Effects of Subgrade Stiffness

The effects of subgrade stiffness on the structural response of skewed jointed concrete pavements were evaluated for a load of 40 kips, either single or tandem axle loads. In addtion, temperature differentials of -10, -5, 0, +8, +18, and +25 ^{o}F in the slab were used with the following input parameters:

1) Slab length = 22 feet, width = 12 feet, thickness = 9 inch
2) Concrete Ec = 5290 ksi, Poisson's ratio = 0.2
3) Joint stiffness:
    a) Rotational stiffness: Lower limit = 1000 k
       Higher limit= 10000 k
    b) Linear stiffness: 10, 200, 750, 1500 ksi
4) Subgrade stiffness:
    a) For soft subgrade = 0.1 kci
    b) For medium subgrade = 0.3 kci
    c) For hard subgrade = 1.2 (Econocrete) kci
5) Live load: Equivalent Joint Edge loading (EJE)
   Equivalent Mid-slab Edge loading (EME)

7.5.1 Initial Deflections and Stresses

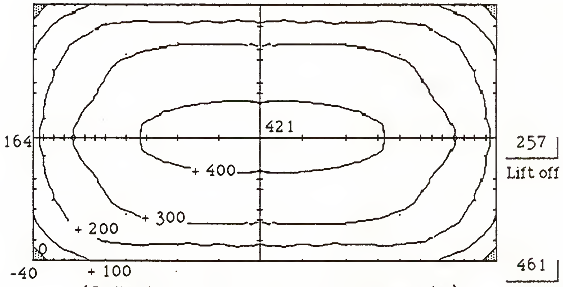
First, temperature differentials of -10 ^{o}F and +25 ^{o}F will be considered. The effects of subgrade modulus on initial deflections and stresses were evaluated. The initial deflection profile in the case of a -10 ^{o}F temperature differential shows that higher subgrade stiffness creates

maximum deflections at the quarter point of a slab from both ends, while forming a W-shaped profile of deflections along the longitudinal center line as shown in Figure 7.8. In addition, Figure 7.9 shows that, with a -10 ^{o}F temperature differential, higher principal stresses exist closer to the corner when the subgrade modulus of subbases stiffens.

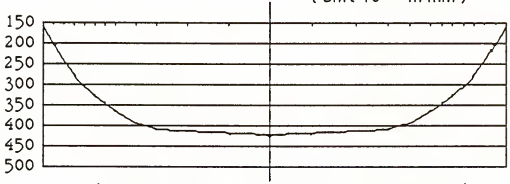
Second, for a +25 ^{o}F temperature differential case, maximum initial deflection at the slab center seems to peak and decrease regardless of joint stiffness, then gradually approach to the soft dome shape of initial deflection profiles by stiffening the subgrade modulus as shown in Figure 7.10. As a result, higher subgrade stiffness has a t endency to create parallel stress profile along the transverse slab center line, which assumes more of a plate pattern in the slab center and also more stress concentration at the joint and edge corners as shown in Figure 7.11, which seems to reflect current corner cracking problems in both -10 ^{o}F and +25 ^{o}F cases.

It is also informative to examine in detail the variations of deflection of several points in the pavement as subgrade stiffens. Table 7.4 and 7.5 and Figure 7.12, 7.13, 7.14 will be used to accomplish this. One of the noticeable phenomena of initial deflections subjected to +25 ^{o}F temperature differential is that the uplift deflection at the slab center does not continuously increase for higher

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1 (Soft)
Joint Stiffness Kr=1000 , Kl=10
Temperature differential = -10 °F



(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



(Deflection along the Longitudinal center line)

(a) ON SOFT SUBGRADE
FIGURE 7.8 INITIAL DEFLECTION PROFILE DUE TO dT=-10 °F

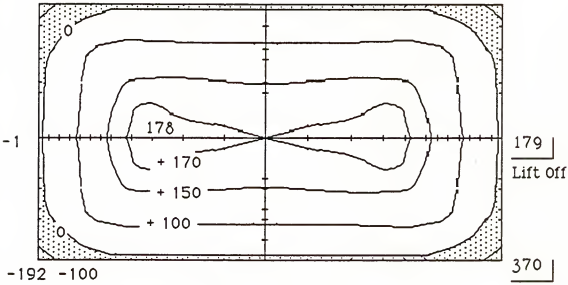
Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°

Material Property Ec = 5290
PR = 0.2

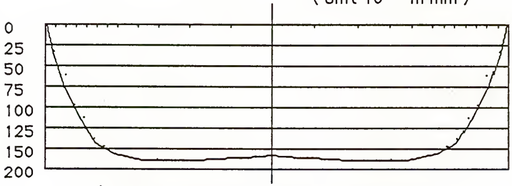
Subgrade Stiffness Ks = 0.3 (Medium)

Joint Stiffness Kr=1000 , Kl=10

Temperature Differential = -10 °F



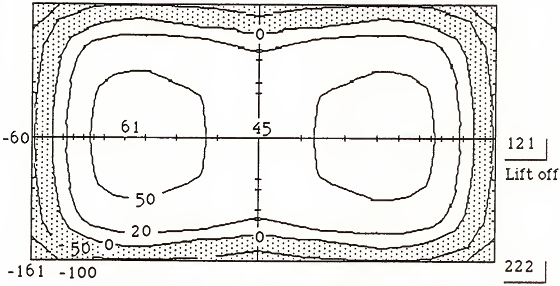
(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



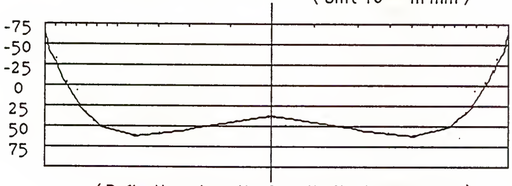
(Deflection along the Longitudinal center line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.8--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks =1.2 (Hard)
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F



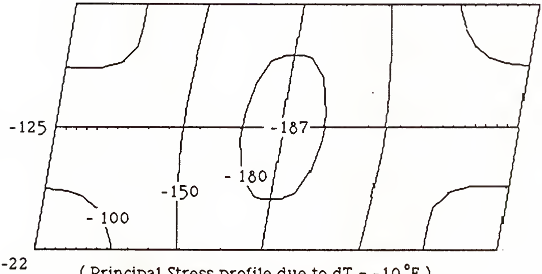
(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



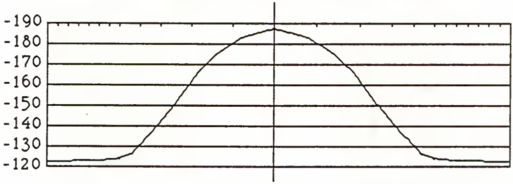
(Deflection along the Longitudinal center line)

(c) ON HARD SUBGRADE
FIGURE 7.8--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F



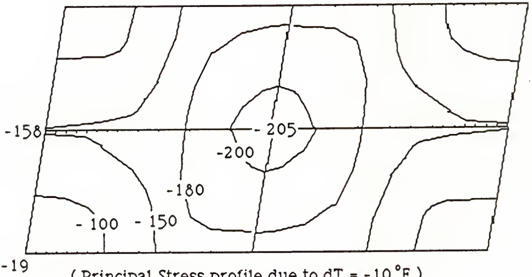
(Principal Stress profile due to dT = -10 °F)
(– sign denotes compression at the bottom in psi)



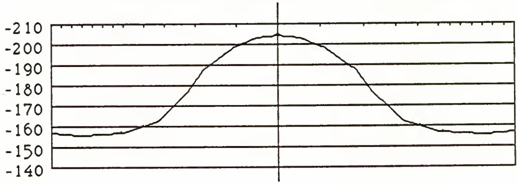
(Stress along the longitudinal center line)

(a) ON SOFT SUBGRADE
FIGURE 7.9 PRINCIPAL STRESS PROFILE DUE TO dT=-10 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0. 3
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F



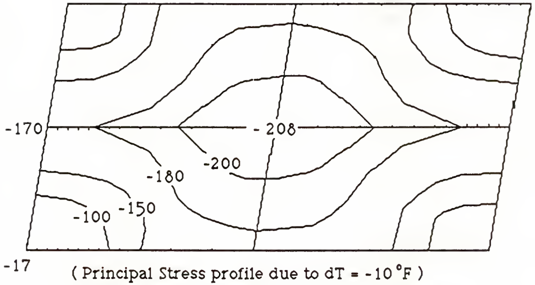
(Principal Stress profile due to dT = -10 °F)
(– sign denotes compression at the bottom in psi)



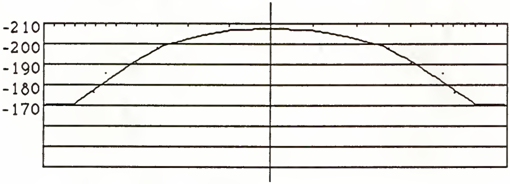
(Stress along the longitudinal center line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.9--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F



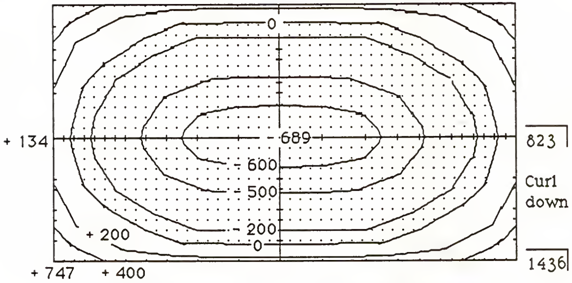
(Principal Stress profile due to dT = -10 °F)
(– sign denotes compression at the bottom in psi)



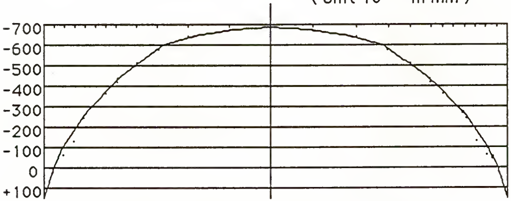
(Stress along the longitudinal center line)

(c) ON HARD SUBGRADE
FIGURE 7.9--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1 (Soft)
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F



(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)

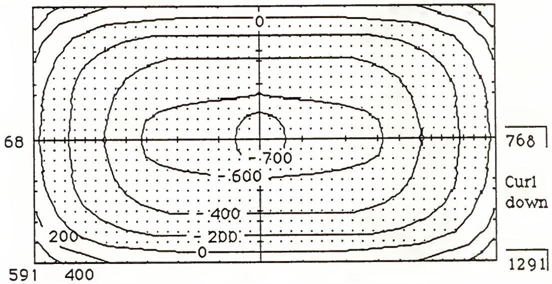


(Deflection along the Longitudinal center line)

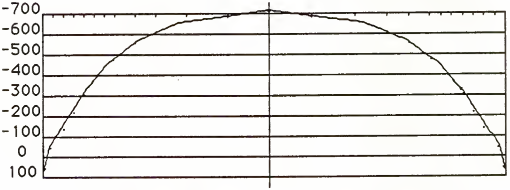
(a) ON SOFT SUBGRADE

FIGURE 7.10 INITIAL DEFLECTION PROFILE DUE TO dT= +25 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.3 (Medium)
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F



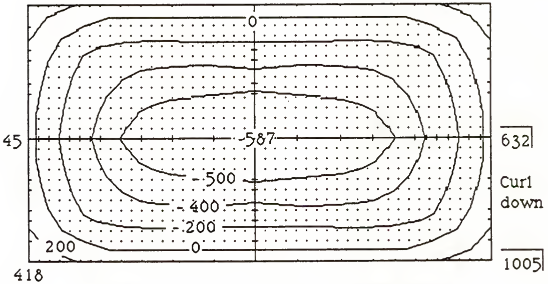
(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



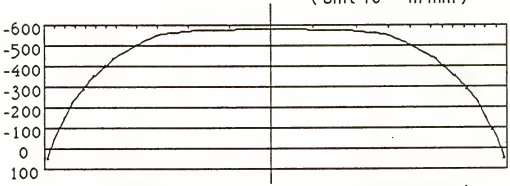
(Deflection along the Longitudinal center line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.10--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2 (Hard)
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F



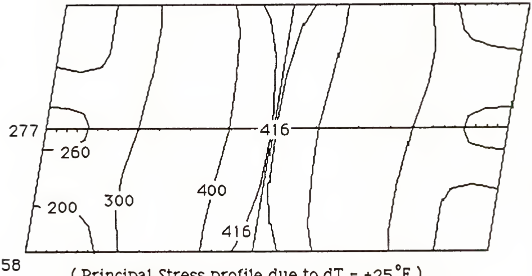
(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



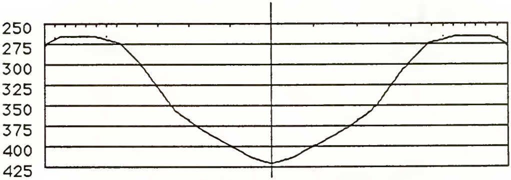
(Deflection along the Longitudinal center line)

(c) ON HARD SUBGRADE
FIGURE 7.10--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = + 25° F



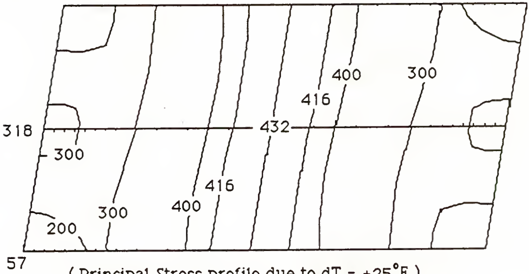
(Principal Stress profile due to dT = +25°F)
(– sign denotes compression at the bottom in psi)



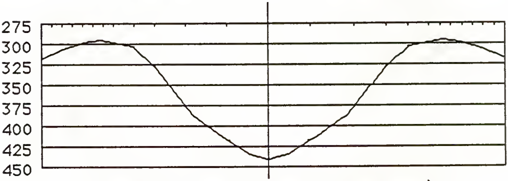
(Stress along the longitudinal center line)

(a) ON SOFT SUBGRADE
FIGURE 7.11 PRINCIPAL STRESS PROFILE DUE TO dT=+25 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.3
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F



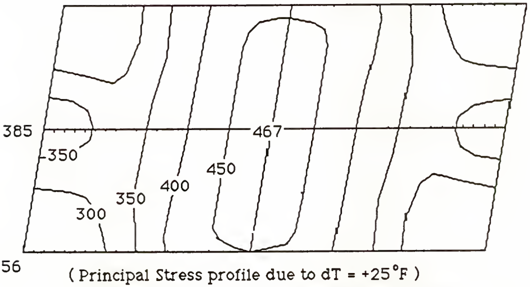
(Principal Stress profile due to dT = +25°F)
(– sign denotes compression at the bottom in psi)



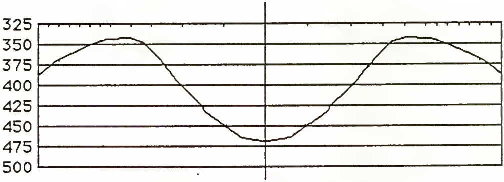
(Stress along the longitudinal center line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.11--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F



(Principal Stress profile due to dT = +25°F)
(+ sign denotes tension at the bottom in psi)



(Stress along the longitudinal center line)

(c) ON HARD SUBGRADE
FIGURE 7.11--continued

Table 7.4 COMPARISON OF INITIAL DEFLECTIONS AND STRESSES ON DIFFERENT SUBGRADES WITH dT = +25 ^{o}F

Subgrade Ks	Kl / Kr	Initial Deflections in 10 (-3)mm: Slab center		Joint center		Edge corner		σ1 psi
	Kl	10	1500	10	1500	10	1500	10
	Kr	1000	10000	1000	10000	1000	10000	1000
Soft 0.1 kci		-689	-531	134	-295	747	473	416
Medium 0.3 kci		-700	-645	68	-346	591	364	432
Hard 1.2 kci		-587	-500	45	-218	418	289	467

Input data : Slab L=22 ft, W=12 ft, Thk=9 in, Skew=9.46
Reference Figures : 7.10 and 7.11.

Table 7.5 COMPARISON OF INITIAL DEFLECTIONS AND STRESSES ON DIFFERENT SUBGRADES WITH dT = -10 ^{o}F

Subgrade Ks	Kl / Kr	Initial Deflections in 10 (-3)mm: Slab center		Joint center		Edge corner		σ1 psi
	Kl	10	750	10	750	10	750	10
	Kr	1000	10000	1000	10000	1000	10000	1000
Soft 0.1 kci		421	394	164	309	-40	56	-187
Medium 0.3 kci		170	161	-1	96	-192	-20	-205
Hard 1.2 kci		45	45	-60	8	-161	-103	-208

Input data : Slab L=22 ft, W=12 ft, Thk=9 in, Skew=9.46
Reference Figures : Fig. 7.8 and Fig. 7.9.

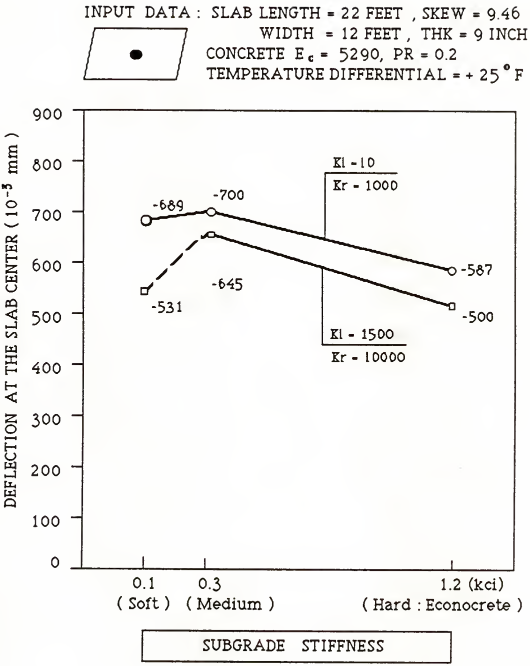


FIGURE 7.12 INITIAL DEFLECTION AT THE SLAB CENTER VS SUBGRADE MODULUS

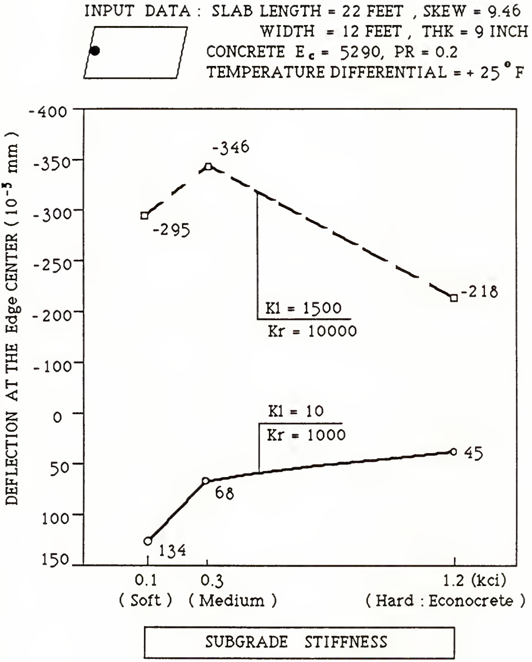


FIGURE 7.13 INITIAL DEFLECTION AT THE JOINT CENTER VS SUBGRADE MODULUS

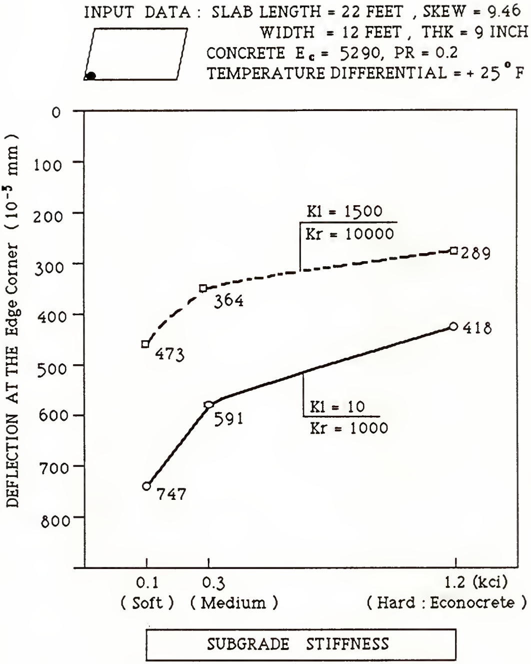


FIGURE 7.14 INITIAL DEFLECTION AT THE EDGE CORNER VS SUBGRADE MODULUS

subgrade stiffness, but actually seems to peak and then decrease regardless of joint stiffness as shown in Figure 7.12. Also, Figure 7.13 and 7.14 verify that initial deflections with same temperature differentials at the joint center and edge corner gradually decrease with higher subgrade stiffness. However, in case of high joint stiffness, for example Kl= 1500 ksi, Kr = 10000 k, the initial deflection at the joint center shows a different curve, which continuously increase and then decrease with a peak point.

On the other hand, Table 7.5 shows that the initial deflections with -10 ^{o}F temperature differential have a tendency to decrease with a higher subgrade stiffness except a corner edge, where uplift actually seems to peak and then decrease. In addition, it is seen that higher joint stiffness does not affect maximum deflection at the slab center with a hard subgrade.

7.5.2 Final Deflections and Stresses.

Next, the effects of subgrade on the slab response under a combination of live load and temperature differential will be evaluated. First of all, initial deflections due to an equivalent joint edge loading of 40 kips and -10 ^{o}F temperature differential are considered.

In Table 7.6, it is seen that the magnitude of maximum up-lift deflections at the slab center and maximum downward deflections at the edge corner are gradually reduced as subgrade stiffens. As a result, it forms more of a soft arch

Table 7.6 EFFECTS OF HIGHER SUBGRADE MODULUS ON FINAL DEFLECTIONS WITH EJE 40 KIPS AND dT=-10 ^{o}F

Subgrade Ks	Final Deflections in 10^{-3} mm Joint corner	Mid-span	σ1 psi
Soft 0.1 kci	833	-28	-307
Medium 0.3 kci	647	-22	-299
Hard 1.2 kci	446	-14	-275

Input data : Slab L=22 ft, W=12 ft, Thk=9 in, Skew=9.46
Joint stiffness Kr=1000 k/in, Kl=10 ksi
Reference Figures : Fig. 7.15 and Fig. 7.16

Table 7.7 EFFECTS OF HIGHER SUBGRADE MODULUS ON FINAL DEFLECTIONS WITH EME 40 KIPS AND dT=+25 ^{o}F

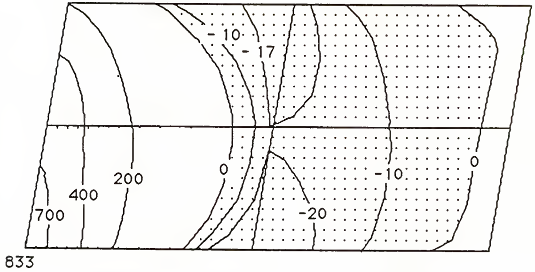
Subgrade Ks	Final Deflections in 10^{-3} mm Joint corner	Slab center	σ1 psi
Soft 0.1 kci	-62	801	860
Medium 0.3 kci	-58	741	842
Hard 1.2 kci	-44	599	853

Input data : Slab L=22 ft, W=12 ft, Thk=9 in, Skew=9.46
Joint stiffness Kr=1000 k, Kl=10 ksi
Reference Figure : Fig. 7.17 and Fig. 7.18

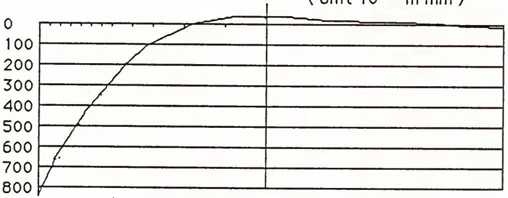
type of patterns as shown in Figure 7.15. Consequently, Figure 7.16 shows that its principal stress profile patterns are significantly changed. It shows more dense stress profiles near joint corners and begins to form corner cracking problems in concrete pavements, even though its magnitudes are slightly reduced as subgrade stiffens. In addition, one of interesting results is that maximum principal stress with joint edge load and -10 ^{o}F temperature differential occurs about 4 feet away from the loading positions, and it is getting closer to joint corner as subgrade stiffens.

On the other hand, for an equivalent mid-slab edge loading of 40 kips and +25 ^{o}F temperature differential, higher subgrade modulus gradually reduces the magnitude of maximum deflection at the slab center and maximum up-lift deflection at the edge corner as shown in Table 7.7, then it forms more of a shallow bowl type of deflection profiles as shown in Figure 7.17. As a result, maximum principal stress at the slab center (one wheel load position) is slightly increased and that of edge center (another wheel load position) is slightly decreased as shown in Figure 7.18. Therefore, it can be noted that the higher subgrade modulus in concrete pavements does not change the magnitude of maximum principal stress that much but it occurs near mid-span with a beam type of stress profile patterns, even though the equivalent single axle load of vehicles

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F
Wheel Load = Joint Edge 40 kip



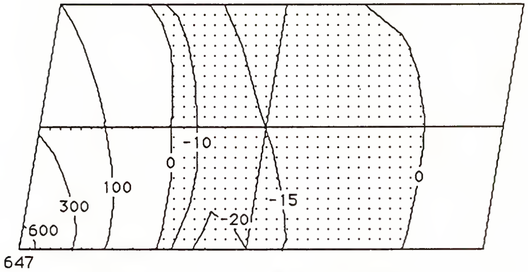
(Deflection Profile due to EJE 40 Kips)
(Unit 10^{-3} in mm)



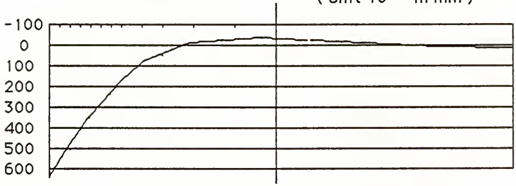
(Deflection along the bottom edge line)

(a) ON SOFT SUBGRADE
FIGURE 7.15 DEFLECTION PROFILE DUE TO EJE 40 KIPS
AND dT= -10 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0. 3
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F
Wheel Load = Joint Edge 40 kip



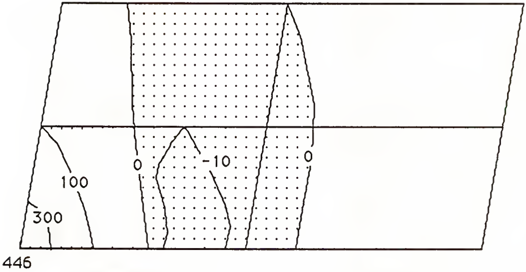
(Deflection Profile due to EJE 40 Kips)
(Unit 10^{-3} in mm)



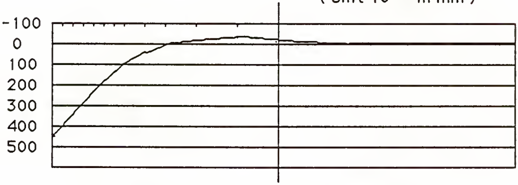
(Deflection along the bottom edge line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.15--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F
Wheel Load = Joint Edge 40 kip



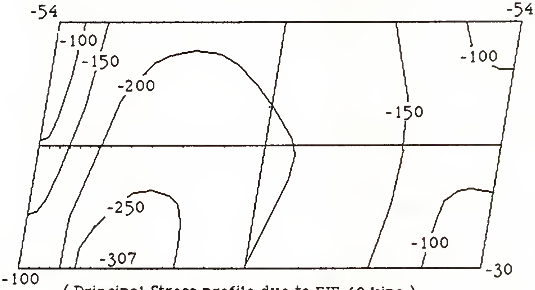
(Deflection Profile due to EJE 40 Kips)
(Unit 10^{-3} in mm)



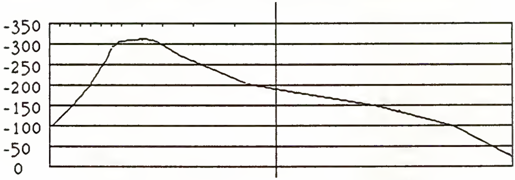
(Deflection along the bottom edge line)

(c) ON HARD SUBGRADE
FIGURE 7.15--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F
Wheel Load = EJE 40 kips



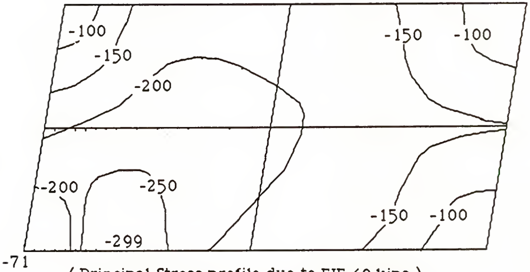
(Principal Stress profile due to EJE 40 kips)
(– sign denotes compression at the bottom in psi)



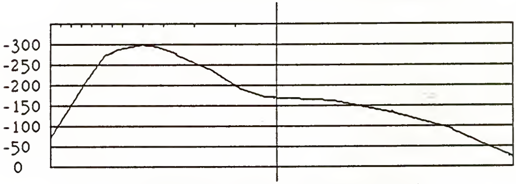
(Stress along the bottom edge line)

(a) ON SOFT SUBGRADE
FIGURE 7.16 PRINCIPAL STRESS PROFILE DUE TO EJE 40 KIPS AND dT= -10 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0. 3
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F
Wheel Load = EJE 40 kips



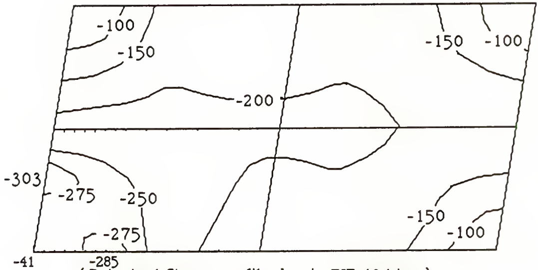
(Principal Stress profile due to EJE 40 kips)
(– sign denotes compression at the bottom in psi)



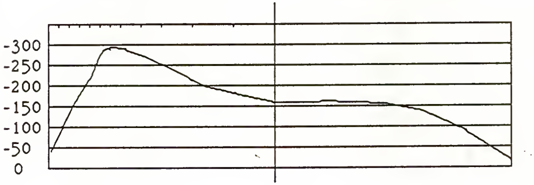
(Stress along the bottom edge line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.16--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = -10 °F
Wheel Load = EJE 40 kips



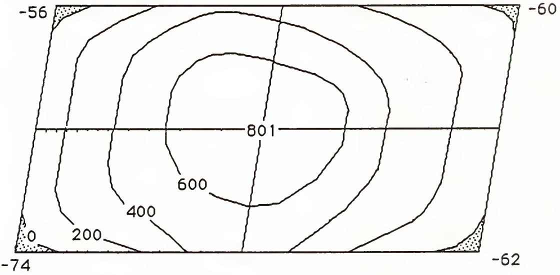
(Principal Stress profile due to EJE 40 kips)
(– sign denotes compression at the bottom in psi)



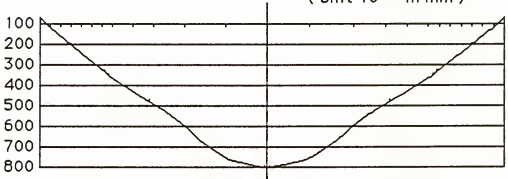
(Stress along the bottom edge line)

(c) ON HARD SUBGRADE
FIGURE 7.16--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = Mid-slab edge 40 kip



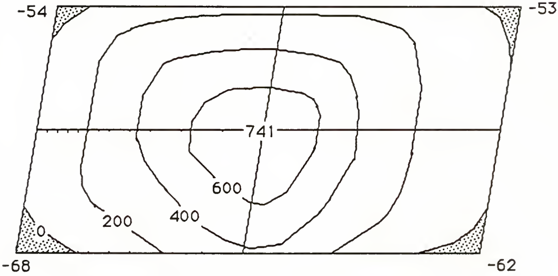
(Deflection Profile due to EME 40 Kips)
(Unit 10^{-3} in mm)



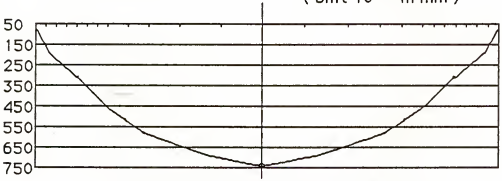
(Deflection along the Longitudinal center line)

(a) ON SOFT SUBGRADE
FIGURE 7.17 DEFLECTION PROFILE DUE TO EME 40 KIPS AND dT= +25 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0. 3
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = Mid-slab edge 40 kip



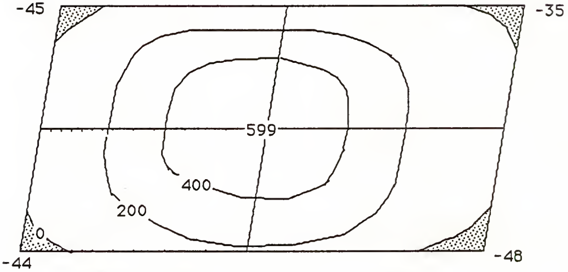
(Deflection Profile due to EME 40 Kips)
(Unit 10^{-3} in mm)



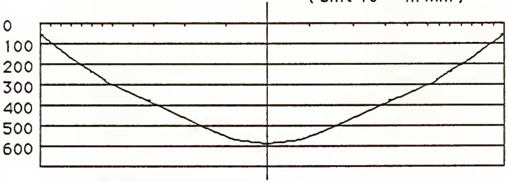
(Deflection along the bottom edge line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.17--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = Mid-slab edge 40 kip



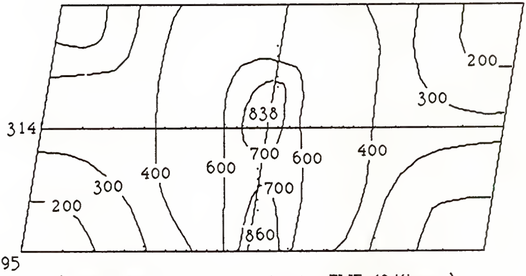
(Deflection Profile due to EME 40 Kips)
(Unit 10^{-3} in mm)



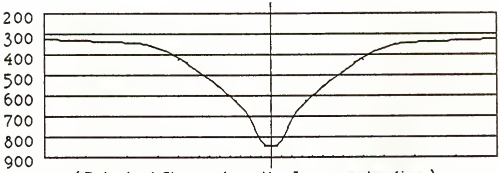
(Deflection along the Longitudinal center line)

(c) ON HARD SUBGRADE
FIGURE 7.17--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = EME 40 kips



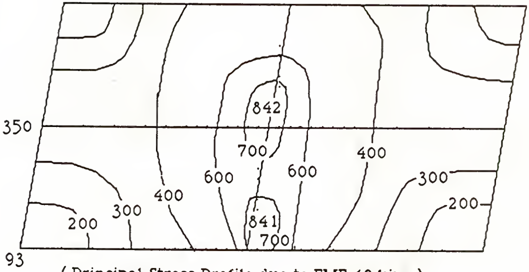
(Principal Stress Profile due to EME 40 Kips)
(Unit in psi)



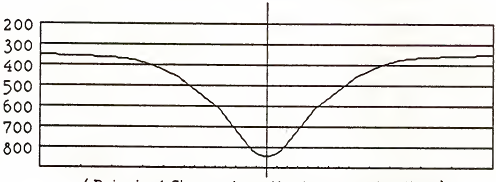
(Principal Stress along the Long. center line)

(a) ON SOFT SUBGRADE
FIGURE 7.18 PRINCIPAL STRESS PROFILE DUE TO EME 40 KIPS AND dT= +25 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.3
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = EME 40 kips



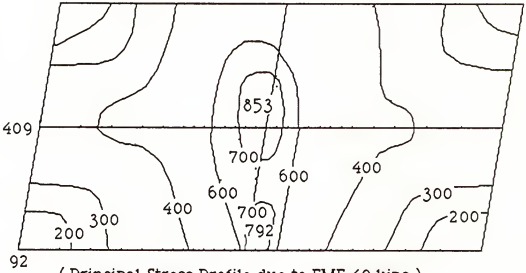
(Principal Stress Profile due to EME 40 kips)
(Units in psi)



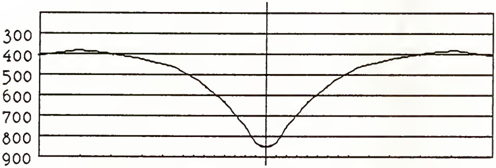
(Principal Stress along the long. center line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.18--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = EME 40 kips



(Principal Stress Profile due to EME 40 kips)



(Principal Stress along the long. center line)

(c) ON HARD SUBGRADE
FIGURE 7.18--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°

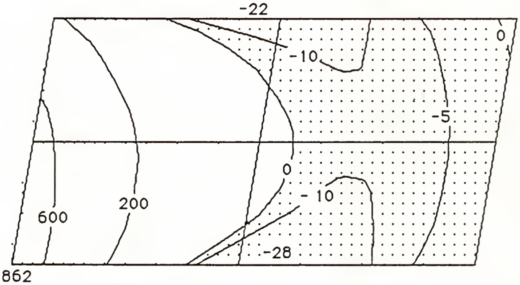
Material Property Ec = 5290
PR = 0.2

Subgrade Stiffness Ks = 0.1

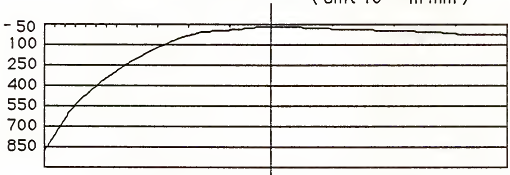
Joint Stiffness Kr=1000 , Kl=10

Temperature Differential = +25 °F

Wheel Load = Joint Edge 40 kip



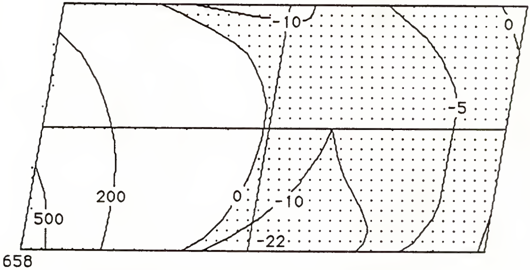
(Deflection Profile due to EJE 40 Kips)
(Unit 10^{-3} in mm)



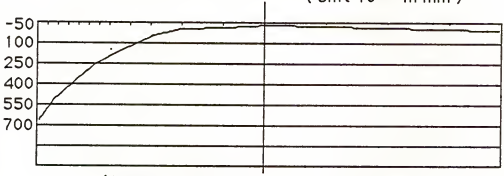
(Deflection along the bottom edge line)

(a) ON SOFT SUBGRADE
FIGURE 7.19 DEFLECTION PROFILE DUE TO EJE 40 KIPS AND dT= +25 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0. 3
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = Joint Edge 40 kip



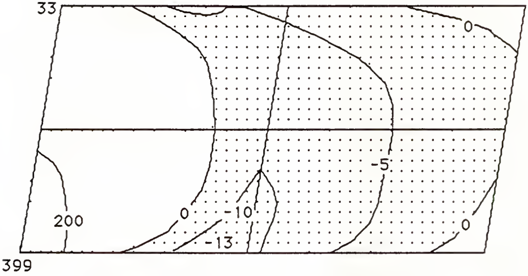
(Deflection Profile due to EJE 40 Kips)
(Unit 10^{-3} in mm)



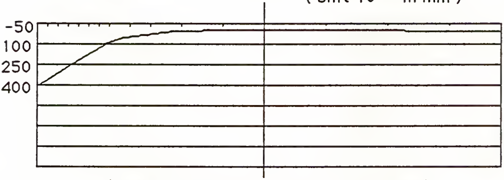
(Deflection along the bottom edge line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.19--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = Joint Edge 40 kip



(Deflection Profile due to EJE 40 Kips)
(Unit 10^{-3} in mm)



(Deflection along the bottom edge line)

(c) ON HARD SUBGRADE
FIGURE 7.19--continued

is placed at the joint-corner of concrete slabs as shown in Figure 7.19 and Fig. 7.20. Again, corner crack problem is visible in those figures.

One of possible loading cases is the joint edge loadings with +25 ^{o}F temperature differential. Table 7.8 shows a usual type of decrease in the maximum deflections and increase in the maximum principal stresses as subgrade stiffened. In comparison with Table 7.6, 7.7, and 7.8, it is seen that maximum downward deflection of edge corner occurs with a combination of joint edge load and +25 ^{o}F temperature differential rather than mid-slab edge load with +25 ^{o}F temperature differential except that of high subgrade modulus.

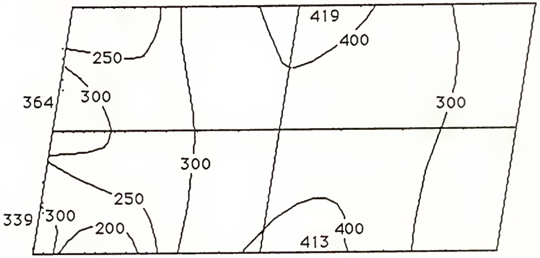
Table 7.8 EFFECTS OF HIGHER SUBGRADE MODULUS ON FINAL DEFLECTIONS WITH EJE 40 KIPS AND dT=+25 ^{o}F

Subgrade Ks	Final Deflections in $10^{(-3)}$ mm		σ1 psi
	Joint corner	Mid-span	
Soft 0.1 kci	862	-28	419
Medium 0.3 kci	658	-22	427
Hard 1.2 kci	399	-13	450

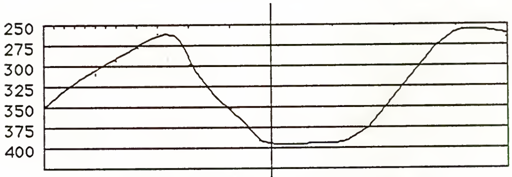
Input data : Slab L=22 ft, W=12 ft, Thk=9 in, Skew=9.46
Joint stiffness Kr=1000 k/in, Kl=10 ksi

Reference Figures : Fig. 7.19 and Fig. 7.20

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = Joint Edge 40 kip



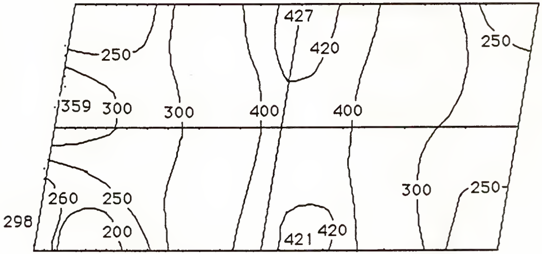
(Principal Stress Profile due to EJE 40 kips)



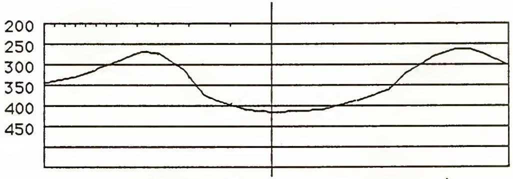
(Principal Stress along the long. center line)

(a) ON SOFT SUBGRADE
FIGURE 7.20 PRINCIPAL STRESS PROFILE DUE TO EJE 40 KIPS AND dT= +25 °F

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.3
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = Joint Edge 40 kip



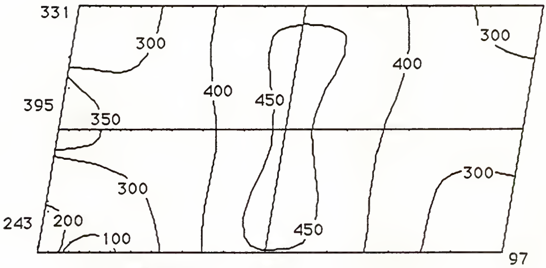
(Principal Stress Profile due to EJE 40 kips)



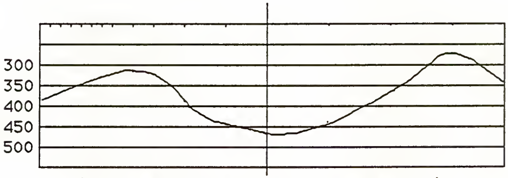
(Principal Stress along the long. center line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.20--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2
Joint Stiffness Kr=1000 , Kl=10
Temperature Differential = +25 °F
Wheel Load = Joint Edge 40 kip



(Principal Stress Profile due to EJE 40 kips)



(Principal Stress along the long. center line)

(c) ON HARD SUBGRADE
FIGURE 7.20--continued

Tables A.1 through H.6 in Appendix A show that the maximum longitudinal stress and the maximum transverse stress in case of zero temperature differential have a tendency to decrease 10 to 20 % with an increase of 0.1, 0.3, and 1.2 kci in subgrade stiffness. However, for -10 or +25 temperature differential, there is very little changes (0 to 4 %) in the computed stress.

Even though we expect more lift-off at the joint edge due to the concave up phenomena of negative temperature differentials, in fact, relative vertical displacements of concrete pavement is reduced from 0.46 millimeter to 0.22 millimeter by stiffening the subgrade modulus as shown in Figure 7.21. And one noticeable fact is that a very stiff subgrade does not cause the slab to be more out of contact when a slab is curled downward at the center due to a positive temperature differential as shown in Table 7.4, even though we generally expected that a very stiff subgrade modulus would cause the slab to be more lift-off at the center, so that relatively low axle loads could destroy it.

Principal stress profiles of mid-slab edge loading with a positive temperature differential of +25 ^{o}F indicate that higher subgrade modulus does not affect the magnitude of principal stress as shown in Table 7.7, but it shows a little change in stress profile patterns as shown in Figure 7.18.

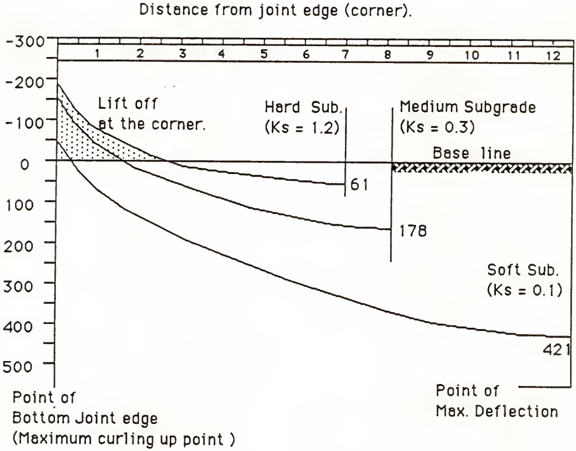


FIGURE 7.21 CANTILEVER EFFECTS OF INITIAL DEFLECTION CURVES ON DIFFERENT SUBGRADE STIFFNESS

In addtion, we can observe more stress concentration on the acute joints by stiffening subgrades in the principal stress profiles, which might cause corner cracking failures in the I-75 skewed concrete pavement. Summary of higher subgrade modulus effects on concrete pavement is shown in Table 7.9, 7.10, 7.11, and 7.12.

In conclusion, it can be noted that the use of high subgrade modulus , say econocrete, is not beneficial to the structural performance of Florida highway concrete pavement, which generally exposed to high temperature differentials. But, for some other areas which have very little or mild temperature differentials, some advantages of econocrete sublayers in the concrete pavement are expected.

7.6 Effects of Joint Stiffness

The effects of joint stiffness were evaluated with the same input parameters:

1) Slab length = 22 feet, width = 12 feet, thickness = 9 inch
2) Concrete Ec = 5290 ksi, Poisson's ratio = 0.2
3) Joint stiffness: 6 sets

    a) Rotational stiffness: Lower limit = 1000 k
    Higher limit= 10000 k

    b) Linear stiffness :10, 200, 750, 1500 ksi

Kr:	1000	1000	1000	10000	10000	10000
Kl:	10	200	750	200	750	1500

Table 7.9 EFFECTS OF HIGHER SUBGRADE MODULUS ON INITIAL DEFLECTIONS

Initial Deflection due to dT = -10 ^{o}F	Initial Deflection due to dT = +25 ^{o}F
i) Edge corner: a) with low joint stiffness (Kl=10, Kr=1000) --Seems to peak and decrease b) with high joint stiffness --gradually increase (more lift off) ii) Other area in the slab: --gradually decrease, and move its maximum point from slab center to a quarter point. (W-type of two domes)	i) Slab center: seems to peak and decrease regardless of joint stiffness ii) Joint center: a) with low joint stiffness (Kr=1000, Kl=10) --gradually decrease b) with high joint stiffness (Kr=10000, Kl=1500) --seems to peak and decrease iii) Edge corner: --gradually decrease iv) Other area: --gradually approach to the soft dome shape

Table 7.10 EFFECTS OF HIGHER SUBGRADE MODULUS ON INITIAL PRINCIPAL STRESS

Initial Principal Stress due to dT = -10 ^{o}F	Initial Principal Stress due to dT = +25 ^{o}F
i) Maximum $\sigma 1$: slightly increase but does not change that much ii) Stress profile pattern: region of higher stresses moves closer to the corner and aligns as for a corner crack	i) Maximum $\sigma 1$: slightly increase but does not change that much ii) Stress profile pattern: more of a plate patterns at the mid-slab and also more stress concentration at the joint and edge corners

Table 7.11 EFFECTS OF HIGHER SUBGRADE MODULUS ON FINAL DEFLECTIONS

Final Deflections due to dT = -10 ^oF and EJE 40 kips	Final Deflections due to dT = +25 ^oF and EME 40 kips
--gradually reduce the magnitude of maximum uplift deflections at the slab center and maximum downward deflections at the edge corner, then forms more of a soft arch type profile	--gradually reduce the magnitude of maximum downward deflection at the slab center and maximum uplift deflection at the edge corner, then forms more of a shallow bowl type profile

Table 7.12 EFFECTS OF HIGHER SUBGRADE MODULUS ON FINAL PRINCIPAL STRESS

Final Principal Stress due to dT = -10 ^oF and EJE 40 kips	Final Principal Stress due to dT = +25 ^oF and EME 40 kips
i) Maximum $\sigma 1$: it occurs about 4 ft away from the joint edge corner and slightly decrease its magnitude.	i) Maximum $\sigma 1$: slightly increase at the slab center and decrease at the edge center
ii) Stress profile pattern: change its stress profile pattern significantly, which show more dense stress profiles at the edge corner and begin to form corner crack problems	ii) Stress profile pattern: general plate pattern of stress profiles at mid-slab and region of higher stresses moves closer to the corner and aligns as for a corner cracks

4) Subgrade stiffness:

a) For soft subgrade = 0.1 kci

b) For medium subgrade = 0.3 kci

c) For Hard subgrade = 1.2 (Econocrete)

5) Live load: Equivalent Joint Edge loading (EJE)

Equivalent Mid-slab Edge loading (EME)

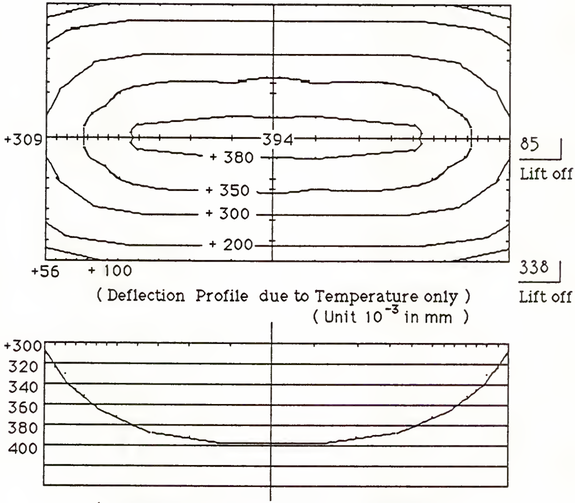
7.6.1 Initial Deflections and Stresses

First, initial deflection profiles with different joint stiffnesses are compared. As shown in Table 7.4 and 7.5, higher joint stiffness shows less deflections in a slab compared to those of lower joint stiffness, especially at the joint center regions, which makes an initial dome shape of deflection profiles transform to the "barrel arch" type of profiles, which curl less-downward at the center of joint with -10 ^{o}F and +25 ^{o}F temperature differentials as shown in Figure 7.22 and 7.23 compared to Figure 7.8 and 7.10. This might be due to the fact that higher joint stiffness induces more restraining action at the joint and approaches a certain "barrel arch" as joint stiffens

7.6.2 Final Deflections and Stresses

Tables A.1 through H.6 in Appendix A present the maximum computed longitudinal and transverse flextural stresses and maximum principal stresses due to four different axle load positions, Joint Edge (JE), Joint Center (JC), Mid-slab Edge (ME), Mid-slab Center (MC) for the various combinations of subgrade stiffnesses and joint stiffnesses.

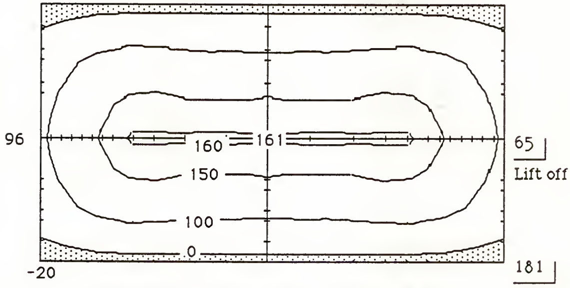
Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1 (Soft)
Joint Stiffness Kr=10000, Kl= 750
Temperature Differential = -10 °F



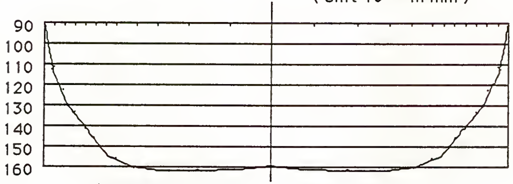
(Deflection along the Longitudinal center line)

(a) ON SOFT SUBGRADE
FIGURE 7.22 INITIAL DEFLECTION PROFILE DUE TO dT=-10 °F WITH HIGHER JOINT STIFFNESS

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.3 (Medium)
Joint Stiffness Kr=10000,Kl= 750
Temperature Differential = -10 °F



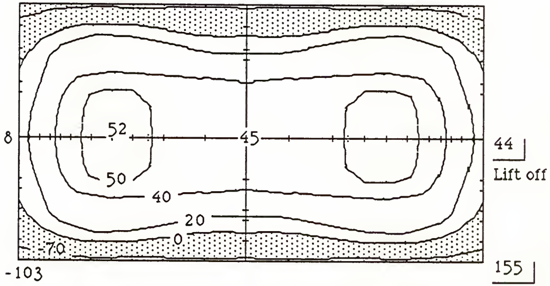
(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



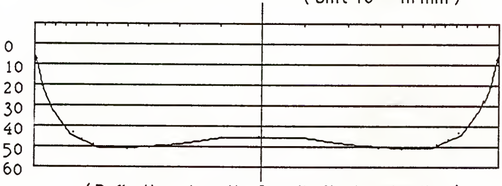
(Deflection along the Longitudinal center line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.22--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2 (Hard)
Joint Stiffness Kr=10000 Kl= 750
Temperature Differential = −10 °F



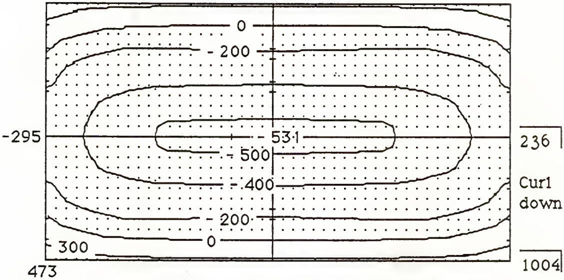
(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



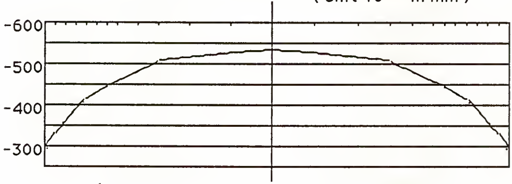
(Deflection along the Longitudinal center line)

(c) ON HARD SUBGRADE
FIGURE 7.22--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.1 (Soft)
Joint Stiffness Kr=10000, Kl=1500
Temperature Differential = + 25° F



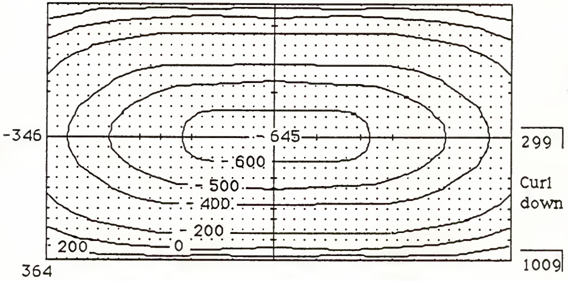
(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



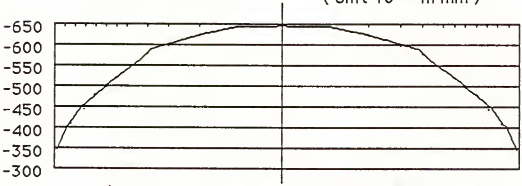
(Deflection along the Longitudinal center line)

(a) ON SOFT SUBGRADE
FIGURE 7.23 INITIAL DEFLECTION PROFILE DUE TO dT=+25 °F
WITH HIGHER JOINT STIFFNESS

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 0.3 (Medium)
Joint Stiffness Kr=10000 Kl=1500
Temperature Differential = + 25 °F



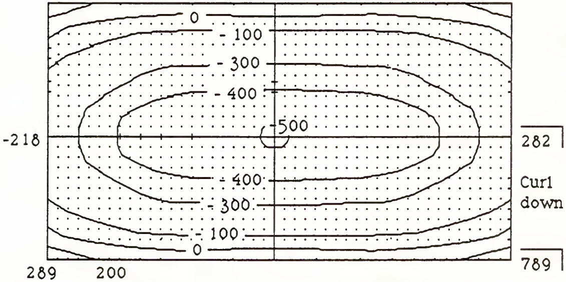
(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



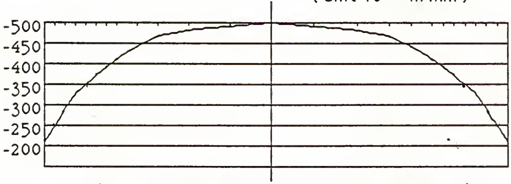
(Deflection along the Longitudinal center line)

(b) ON MEDIUM SUBGRADE
FIGURE 7.23--continued

Input Data : Slab Dimension Length=22 ft
Width = 12ft
Thick = 9 in
Skew = 9.46°
Material Property Ec = 5290
PR = 0.2
Subgrade Stiffness Ks = 1.2 (Hard)
Joint Stiffness Kr=10000 Kl=1500
Temperature Differential = + 25 °F



(Deflection Profile due to Temperature only)
(Unit 10^{-3} in mm)



(Deflection along the Longitudinal center line)

(c) ON HARD SUBGRADE
FIGURE 7.23--continued

It is obvious that with joint edge loadings deflections were far more sensitive to variation in Kl values. Increasing Kl values with a low fixed Kr values results in the decrease of maximum principal stress on the loaded slab, while increasing Kl values with a high fixed Kr values does not affect maximum principal stress significantly. On the other hand, mid-slab loadings has shown no effects on both flexural and shear stress with variations of joint stiffness. It seems that joint stiffness affect the structural performance of joint loading case only, and does not seriously affect mid-slab loading case. Besides, joint stiffness is never important when maximum principal stress occurs away from the joint. This is interesting as one would expect the uplifted corner of a slab to be the maximum stress region and be very sensitive to joint stiffness.

Table 7.13 and Figure 7.24 show the maximum longitudinal and transverse stresses to see how those stresses varies with the various combinations of joint stiffnesses. It is seen that higher joint stiffness results in increase of flexural stresses (σx, σy) with a positive temperature differential and decrease of flexural stresses (σx, σy) with a negative temperature differential. On the other hand, with no temperature differential, it shows increase of σx and decrease of σy. From Table, it can be noted that σx is more sensitive to higher joint stiffness rather than σy.

Table 7.13 EFFECTS OF JOINT STIFFNESS ON MAXIMUM STRESS DUE TO EJE 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT (^{o}F) -10		0		+25	
Ks	Kr	Kl	σx	σy	σx	σy	σx	σy
0.1 (S)	Low 1000	10	-307 N	361 L	-209 L	475 L		
		200	-260 N	207 L	177 L	324 L		
		750	-260 N	172 L	181 L	288 L	451 F	574 L
	High 10000	200	-251 N	209 L	321 L	327 L		
		750	-247 N	174 L	324 L	293 L		
		1500			325 L	286 L	669 L	594 L
0.3 (M)	Low 1000	10	-296 N	250 L	-196 L	393 L		
		200	-248 N	-208 C	159 L	255 L		
		750	-243 N	-209 C	164 L	219 L	447 F	563 L
	High 10000	200	-243 N	-211 C	281 L	266 L		
		750	-240 N	-211 C	284 L	231 L		
		1500			285 L	224 L	657 L	605 L
1.2 (H)	Low 1000	10	-281 N	-264 C	-176 L	321 L		
		200	-235 N	-275 C	135 L	213 L		
		750	-234 N	-283 C	141 L	178 L	414 F	583 L
	High 10000	200	-235 N	-283 C	227 L	222 L		
		750	-234 N	-283 C	231 L	189 L		
		1500			231 L	182 L	615 L	644 L

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Ks = Spring stiffness of subgrade.
Kr,Kl = Rotational and Linear spring stiffness
L means that maximum stress occurs at load point,
N : Near load point, C : between two wheel load,
F : Far away from load point

Input Parameters : Slab L=22 ft, W=12 ft, t= 9 in
Concrete Ec=5290, PR=0.2
Subgrade Stiffness = 0.1
Wheel Load = EJE 40 Kips
Skew Angle = 9.46, dT = 0 F

Curve	Joint Stiffness		
	Kl	Kr	Ke
1	10	4000	10
2	10	25000	10
3	1000	32000	10

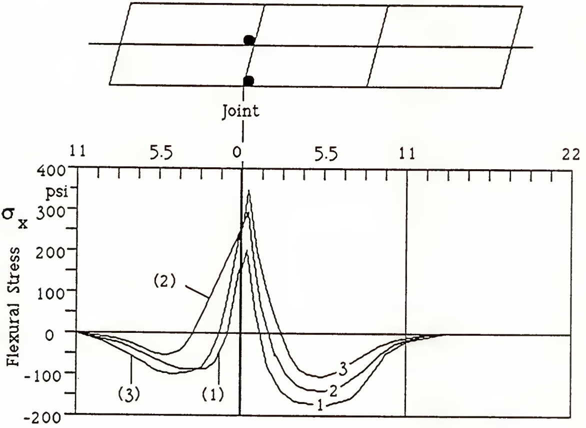


FIGURE 7.24 EFFECTS OF JOINT STIFFNESS ON FLEXURAL STRESS

For the mid-span loading, Table A.3.4 demonstrates that linear and rotational spring stiffness of joint have no effect on the maximum principal stress in both -10 ^{o}F and +25 ^{o}F temperature differential cases, but higher subgrade stiffness results in a decrease of maximum principal stress, which occurs at the mid-span loading points with the beam type of stress profiles.

Another way to investigate the effects of joint stiffness is to draw flexural stress lines as shown in Figure 7.24. It seems that the higher joint stiffness can lead to more stress transfer, but it does not affect overall stress levels.

7.7 Effects of Temperature Differentials

A typical pavement system with the following input parameters was used:

1) Slab length = 22 feet, width = 12 feet, thickness = 9 inch
2) Concrete Ec = 5290 ksi, Poisson's ratio = 0.2
3) Joint stiffness: 6 sets

   a) Rotational stiffness: Lower limit = 1000 k
   Higher limit= 10000 k

   b) Linear stiffness: 10, 200, 750, 1500 ksi

Kr:	1000	1000	1000	10000	10000	10000
Kl:	10	200	750	200	750	1500

4) Subgrade stiffness:

   a) For soft subgrade = 0.1 kci

b) For medium subgrade = 0.3 kci

c) For Hard subgrade = 1.2 (Econocrete)

5) Live load: Equivalent Joint Edge loading (EJE)

Temperature curling may be considered as the main factoror contributing to the concrete pavement failures. In general, temperature differentials (top-bottom) causes the individual slabs to lift either in the center (+) or at the corners (-), depending on the nature of the gradient. This can cause loss of contact between pavement and subgrade.

7.7.1 Initial Stresses and Deflections

Temperature differentials of 0, -10, and +25 ^{o}F will be considered. First, Figure 7.25 shows that there is no initial stress in the slab in case of zero temperature differential. As temperature curling grows, -10 ^{o}F temperature differential creates about -150 psi flexural stress in X-direction (σx) and +20 ^{o}F temperature differential produces about 300 psi flexural stress in X-direction (σx), concave up and down respectively. Table 7.14 also demonstrates that initial flexural stresses of slab center are always greater than those of joint or edge lines as we expected. Therefore, it can be noted that an initial stress due to temperature differential (temperature on the top surface-bottom surface) is almost linearly related to the magnitude of temperature differentials, even though negative temperature differential creates tensile stress at the upper portion of

Input Parameters : Slab L=22 ft, W=12 ft, t= 9 in
Concrete Ec=5290, PR=0.2
Subgrade Stiffness = 0.1
Wheel Load = None
Skew Angle = 9.46

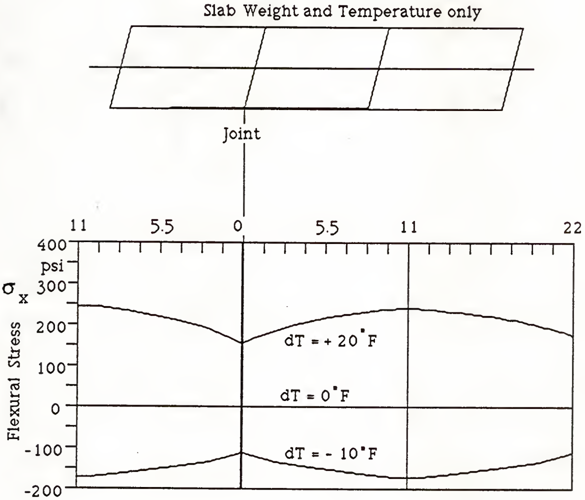


FIGURE 7.25 INITIAL STRESS DUE TO TEMPERATURE DIFFERENTIAL

Table 7.14 LEVEL OF THERMAL INITIAL STRESS FOR VARIOUS SLAB REGIONS

Location Along the	δT ^{o}F	Average σ_x	Stress σ_y
Edge line	0	0	0
	-10	-150 psi	-7 psi
	+20	305 psi	12 psi
Long. Center line	0	0	0
	-10	-178 psi	-145 psi
	+20	345 psi	228
Joint line	0	0	0
	-10	-145 psi	-85 psi
	+20	298 psi	150 psi
Cross at the mid-slab	0	0	0
	-10	-179 psi	-95 psi
	+20	347 psi	130 psi

Input Data : Slab length = 22 ft, Width = 12 ft,
Applied load = Temperature and Dead load only.
Thk = 9 in, Skew = 9.46 ^{o}
Concrete Ec = 5290, Poisson's ratio = 0.2
Stiffness Ks = 0.175, Kl = 10 ksi
Ke = 10 , Kr = 16000 k

slab and positive temperature differential yields tensile stress at the lower portion of slab as general design sign conventions follow. As a matter of fact, it really does not matter where tensile stress occurs as long as it creates maximum critical stress. However, this thermal initial stress usually gets into problems with several different wheel load positions.

7.7.2 Final Deflections and Stresses

This section will discuss the final deflection and stress caused by a temperature differential and axle load. Figure 7.26 shows how thermal gradients affect joint edge loadings. Maximum principal stresses caused by -10 ^{o}F and +25 ^{o}F temperature differentials with joint edge loadings are 274 psi and 804 psi at the loading points as shown in Figure 7.27. Figure 7.27 through 7.30 clearly show that the maximum stress increases with an increase in temperature differentials in the slab regardless of subgrade modulus and wheel load positions. Those figures indicate that maximum principal stress is far more sensitive to Kl variations rather than Kr values, which show very linear relationships between temperature differentials and maximum principal stress except that of low joint stiffness on soft subgrade with negative temperature differentials as shown in Figure 7.27.

7.8 Effects of Wheel Load Positions

Even though most of the vehicles on the highway travel along the longitudinal center line of each lane, there might be a chance of some vehicles traveling along the edge line in some cases. Tables in the Appendix A compare stresses of joint edge loading and mid-slab edge loading with those of joint center loading and mid-slab center loading. Table 7.15 and 7.16 show that maximum flexural stresses of edge loadings

Input Parameters : Slab L=22 ft, W=12 ft, t= 9 in
Concrete Ec=5290, PR=0.2
Subgrade Stiffness = 0.1
Wheel Load = EJE 40 Kips
Skew Angle = 9.46, dT = - 10°F
Joint Stiffness Ke=10, Kl=10, Kr=16000

1/4 Point	Joint	1/4 Point	Stress
-183 psi	-157.8 psi	-180.6 psi	σ (dT+D.L)
-54	205	-44.6	σ (L.L)
-237	47.2	-225.2	σ (dT+D.L) + σ (L.L)
-238	47.4	-226.2	σ (dT+D.L +L.L)

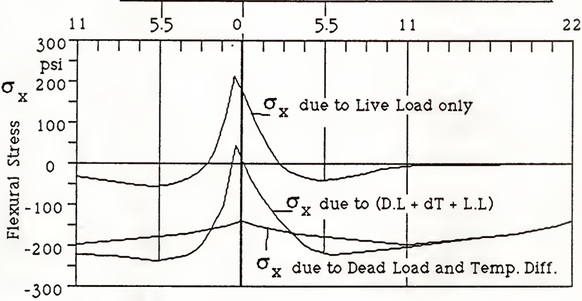


FIGURE 7.26 COMPARISON OF σ(dT+D.L) PLUS σ(L.L) WITH σ(dT+D.L+L.L)

INPUT DATA : SLAB LENGTH = 22 FEET , SKEW = 9.46
WIDTH = 12 FEET , THK = 9 INCH
CONCRETE E_c = 5290, PR = 0.2
SUBGRADE STIFFNESS = 0.1 (SOFT)
AXLE LOAD = EQUI. JOINT EDGE 40 KIPS

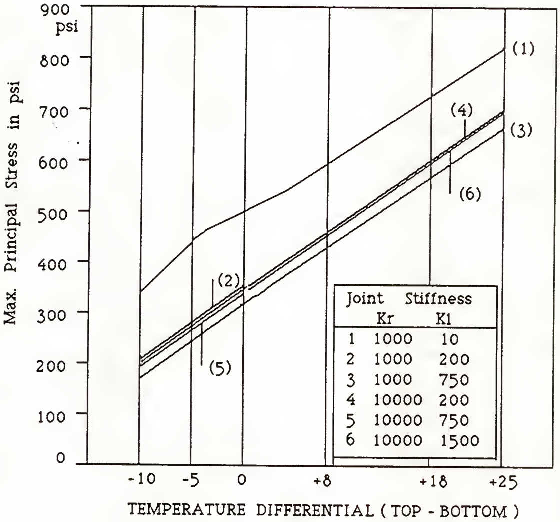


FIGURE 7.27 TEMPERATURE EFFECTS ON $\sigma 1$ DUE TO EJES

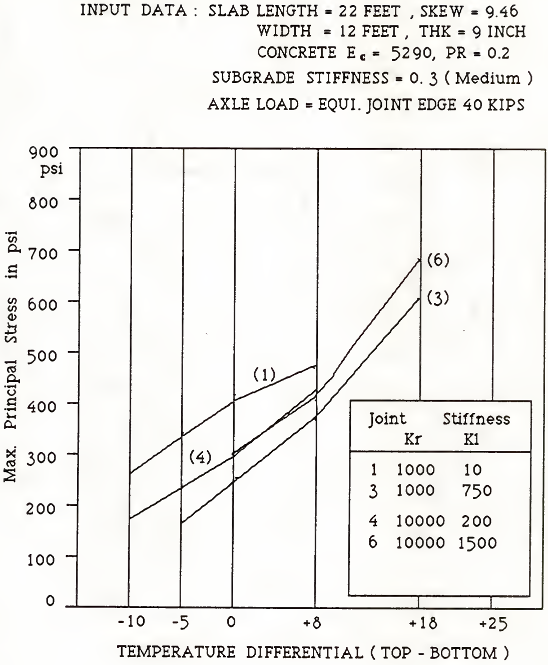


FIGURE 7.28 TEMPERATURE EFFECTS ON $\sigma 1$ DUE TO EJEM

INPUT DATA : SLAB LENGTH = 22 FEET , SKEW = 9.46
WIDTH = 12 FEET , THK = 9 INCH
CONCRETE E_c = 5290, PR = 0.2
SUBGRADE STIFFNESS = 1.2 (Hard)
AXLE LOAD = EQUI. JOINT EDGE 40 KIPS

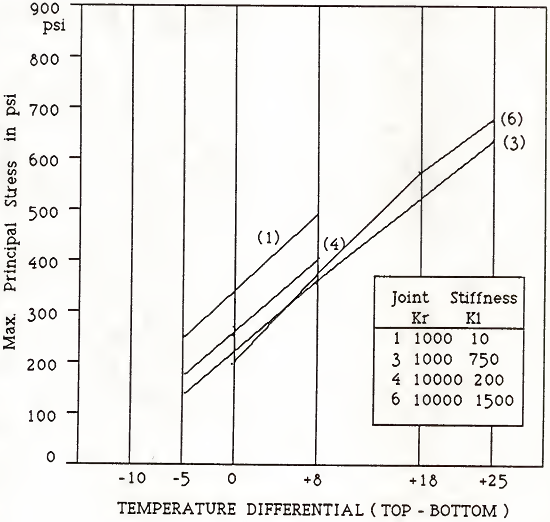


FIGURE 7.29 TEMPERATURE EFFECTS ON $\sigma 1$ DUE TO EJEH

INPUT DATA : SLAB LENGTH = 22 FEET , SKEW = 9.46
WIDTH = 12 FEET , THK = 9 INCH
CONCRETE E_c = 5290, PR = 0.2
AXLE LOAD = Equiv. Mid-Slab Edge 40 kips

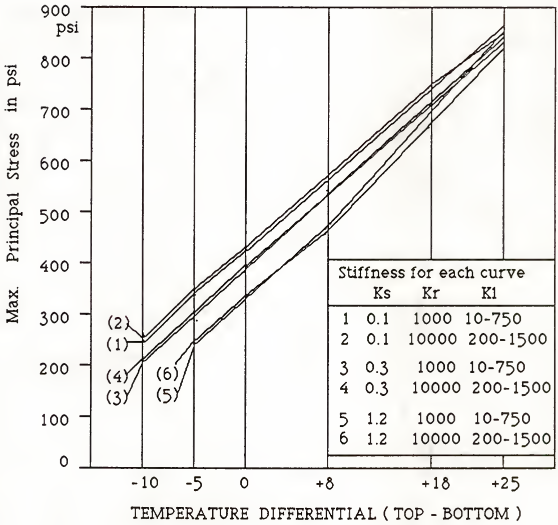


FIGURE 7.30 TEMPERATURE EFFECTS ON $\sigma 1$ DUE TO EME

Table 7.15 COMPARISONS OF JOINT CENTER LOADING AND JOINT EDGE LOADING

Ks	Kr	$\frac{\sigma 1\ EJC}{\sigma 1\ EJE}$	$\frac{\sigma 1\ TJE}{\sigma 1\ EJE}$	$\frac{\sigma 1\ TJC}{\sigma 1\ EJE}$
0.1	Low (1000)	$\frac{460.0}{495.5} = 0.93$	$\frac{202.3}{495.5} = 0.41$	$\frac{186.8}{495.5} = 0.38$
	High (10000)	$\frac{287.8}{325.0} = 0.89$	$\frac{226.2}{325.0} = 0.70$	$\frac{188.6}{325.0} = 0.58$
0.3	Low (1000)	$\frac{378.3}{409.6} = 0.92$	$\frac{203.5}{409.6} = 0.50$	$\frac{137.9}{409.6} = 0.34$
	High (10000)	$\frac{237.3}{284.1} = 0.84$	$\frac{212.2}{284.1} = 0.75$	$\frac{142.1}{284.1} = 0.50$
1.2	Low (1000)	$\frac{301.1}{330.8} = 0.91$	$\frac{198.0}{330.8} = 0.60$	$\frac{108.3}{330.8} = 0.33$
	High (10000)	$\frac{183.0}{231.7} = 0.79$	$\frac{190.5}{231.7} = 0.82$	$\frac{108.9}{231.7} = 0.47$

Table 7.16 COMPARISONS OF MID-SLAB CENTER LOADING AND MID-SLAB EDGE LOADING

Ks	Kr	$\frac{\sigma 1\ EMC}{\sigma 1\ EME}$	$\frac{\sigma 1\ TME}{\sigma 1\ EME}$	$\frac{\sigma 1\ TMC}{\sigma 1\ EME}$
0.1	Low (1000)	$\frac{347.0}{413.5} = 0.84$	$\frac{244.7}{413.5} = 0.59$	$\frac{194.8}{413.5} = 0.47$
	High (10000)	$\frac{347.1}{414.1} = 0.84$	$\frac{244.9}{414.1} = 0.59$	$\frac{193.6}{414.1} = 0.47$
0.3	Low (1000)	$\frac{289.9}{375.9} = 0.77$	$\frac{221.9}{375.9} = 0.59$	$\frac{148.7}{375.9} = 0.40$
	High (10000)	$\frac{290.5}{376.6} = 0.77$	$\frac{222.5}{376.6} = 0.59$	$\frac{149.0}{376.6} = 0.40$
1.2	Low (1000)	$\frac{225.0}{311.7} = 0.60$	$\frac{190.0}{311.7} = 0.61$	$\frac{149.0}{311.7} = 0.36$
	High (10000)	$\frac{225.1}{311.8} = 0.60$	$\frac{190.2}{311.8} = 0.61$	$\frac{111.8}{311.8} = 0.36$

are about 10 to 40 % greater, depending on the subgrade stiffness, than those of center loadings of same magnitude.

The increase in maximum principal stress due to load on the edge (EJE, EME) rather than along the center line (EJC, EMC) is most pronounced for loads near the mid slab and for slabs with higher subgrade modulus. It appears that stiffening the subgrade even makes relatively more destructive to the pavement for trucks to travel along the edge. In addition, Table 7.15 and 7.16 demonstrate that effects of equivalent single axle load produce about 10 to 50 % greater maximum principal stress than that of a tandem axle load, which often used for heavier truck loads. Figure 7.31 shows how this tandem axle load affects flexural stress in the slab. Therefore, it can be noted that an equivalent single axle load generally produces more critical loading conditions rather than the tandem axle load, which provides reasonable backgrounds to the design codes or specifications.

Input Parameters : Slab L=22 ft, W=12 ft, t= 9 in
Concrete Ec=5290, PR=0.2
Subgrade Stiffness = 0.1
Wheel Load = EJE 40 Kips
Skew Angle = 9.46, dT = - 10°F
Joint Stiffness Ke=10, Kl=10, Kr=16000

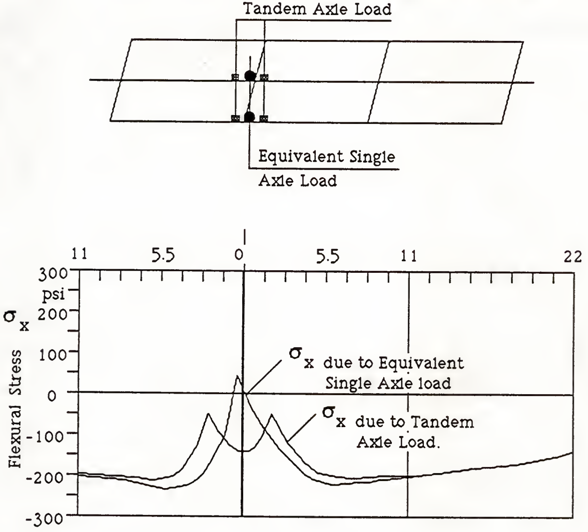


FIGURE 7.31 COMPARISON OF TANDEM AND EQUIVALENT AXLE LOAD ALONG THE BOTTOM EDGE LINE

CHAPTER 8
CONCLUSIONS AND RECOMMENDATIONS

8.1 Summary

An analytical study of response of jointed concrete highway pavements was performed. First, improvements of previous finite element analysis programs, FEACONS and INPLANE, were made to analyze the skewed jointed element. This consisted of derivations of finite element technique for the parallelogram plate bending element and in-plane stress element. The second part of the study involved response analysis of the pavement for various combinations of significant parameters, such as skewed angles, subgrade moduli, joint stiffnesses, temperature differentials, and axle load positions.

The results of those parametric studies were tabulated in Appendix A and were explained in detail in Chapter 7 with even better stress and deflection profiles, which certainly provided a very concise way of looking at the distribution of stresses and the variations of displacements on the surface of the concrete slab. The following sections lead to the conclusions of parametric studies.

8.1.1 Effects of Skewed joints

1) The use of skewed joints in concrete pavement induces slightly higher stress levels. For instance, the major

principal stress due to the joint edge load increases and then decrease with increased skew angles. The minor principal stress increases, but only for the highest skew angle, while the shear stresses increases continually. In no case, however, are the variations in stress truly striking.

2) Even though the variations in maximum stress are not drastic, these higher stress levels are reached closer to the corner when the joint is skewed.

3) Micro-study of stress variations for the corner crack problems showed that the flexural stress in the X- direction due to the 40 kip joint edge load decreases very slightly as skew angle grows. However, the flexural stress in the Y - direction increases as skew angle grows. This result, coupled with the presence of higher principal stresses closer to the joint, may contribute to the higher frequency of corner crackings on the skewed jointed pavements.

8.1.2 Effects of Subgrade Stiffness

The use of econocrete sublayer was suspected to cause more lift-off either at the joint edge or at the slab center, depending on the nature of the themal gradients. However, as observed, in the case of negative temperature differential, a higher subgrade modulus creates maximum initial deflections at the quarter points of a slab from both ends and expands its maximum initial principal stress region to those quarter points. On the other hand, higher subgrade modulus with a positive temperature differential shows that

Table 8.1 SUMMARY OF SUBGRADE MODULUS EFFECTS

Temperature Differential	Initial Deflection	Initial Principal Stress
- 10 °F	Higher Ks-- move maximum defle -ction point from slab center to quarter points.	Higher Ks-- accordingly, change stress profile pattern ,which expands their max. stress profile region to quarter points, but magnitude of max $\sigma 1$ does not change that much.
+ 25 °F	Higher Ks-- Uplift deflection at the slab center does not continuous -ly increase but seems to peak and then decrease.	Higher Ks-- Create parallel stress profile along the transverse slab center line, which shows more of a plate pattern.

	Maximum Loaded Principal Stress
- 10 °F	Equivalent joint edge loading: 1) maximum principal stress occurs about 4 ft away from the joint edge corner. 2) higher subgrade modulus does not affect maximum principal stress, but shows different pattern of stress profiles
+ 25 °F	Joint and Mid-slab edge loading: Higher subgrade modulus does not affect maximum principal stress, but it changes stress profile pattern with more of a beam type of pattern.

initial uplift deflections at the slab center do not continuously increase but actually seem to peak and then decrease, producing more of a plate pattern in the slab center, which might be a possible cause of transverse cracking in the pavement.

Such reduced relative displacements on the high subgrade stiffness of econocrete sublayer decrease 10 to 20 % in the maximum computed flexural stresses with no thermal gradients and also show very little change (0 to 4 %) in the computed stresses with -10 and +25 oF temperature differentials, which means that the use of unbonded econocrete sublayer in Florida is of little benefit to the pavement system.

8.1.3 Effects of Joint Stiffness

1) Higher joint stiffness shows less deflections in a slab compared to those of lower joint stiffness, especially at the joint center regions, which makes an initial dome shape of deflection profiles transform to the "barrel arch" type of profiles, which curl less-downward at the center of joint with -10 ^{o}F and +25 ^{o}F temperature differentials.

2) Deflections were far more sensitive to variation in Kl values. Increasing Kl values with a low fixed Kr values results in the decrease of maximum principal stress on the loaded slab, while increasing Kl values with a high fixed Kr values does not affect maximum principal stress seriously.

3) On the other hand, mid-slab loadings has shown no effects on both flexural and shear stress with variations of joint stiffness. It seems that joint stiffness affect the structural performance of joint loading case more, and does not seriously affect mid-slab loading case.

4) Higher joint stiffness results in increase of flexural stresses (σ_x, σ_y) with a positive temperature differential and decrease of flexural stresses (σ_x, σ_y) with a negative temperature differential. On the other hand, with no temperature differential, it shows increase of σ_x and decrease of σ_y. However, it does not affect overall stress levels.

8.1.4 Effects of Temperature Differentials

An initial stress due to a temperature differential is almost linearly related to the magnitude of the temperature differential. As temperature curling grows, a -10 ^{o}F temperature differential creates about -150 psi flexural stress in X-direction (σ_x) and +20 ^{o}F temperature differential produces about 300 psi flexural stress in X-direction (σ_x). And initial flexural stresses of slab center are always greater than those of joint or edge lines, as we expected.

8.1.5 Effects of Axle Load Positions

1) Maximum principal stress of an equivalent single axle loading is always greater than that of tandem axle loadings.

2) The increase in maximum principal stress due to load on the edge rather than along the center line is most pronounced for loads near the mid slab and for slabs with higher subgrade modulus. It appears that stiffening the subgrade even makes relatively more destructive to the pavement for trucks to travel along the edge.

8.2 Comments on MZC Plate Rectangle

The 12 degree of freedom plate bending element incorporating skewed joint geometry follows that developed by Melosh, Zienkiewicz, and Chung. This element is nonconforming because normal slopes are not compatible, and discontinuities occur at adjoining edges. The normal slopes along an edge varies cubically and is not uniquely defined by two slopes at the ends of the edge.

On the other hand, Bogner, Fox, and Schmit introduced a conforming element, which is called BFS rectangle. It has four degree of freedom, which consist of a vertical displacement, two rotations, and a warping displacement, at each node. This BFS rectangle has normal slope compatibility at all edges. The normal slope along an edge has a cubic variation that is controlled by four parameters, which are the normal slope and warp at each end (Ref.19).

This BFS rectangle generally produces greater accuracy than the MZC rectangle because of a higher order displacement function and a larger number of nodal displacements.

Nevertheless, both MZC and BFS rectangle can model states of constant strain for a flexure in plate, and they also have complete and balanced displacement function, which produces great convergence.

8.3 Recommendations

The following recommendations can be made:

1) The use of high stiffness unbonded subgrade layers were associated in this research with unfavorable distributions of pavement stress. The use of this type of pavement section would seem very questionable.

2) While there are undoubtably advantages in using skewed joints, it should be noted that this geometry can somewhat alter the distribution of pavement stress and produce unfavorable stress conditions at acute corners.

3) In evaluating the stresses in a concrete pavement, it should be remembered that variations in basic pavement parameters, such as temperature differentials, subgrade stiffness, joint stiffness, affect not only maximum stress but the entire distribution of stress, and that these changes in distribution can be important.

4) More research needs to be performed on how joint stiffness, subgrade properties, and temperature differentials affect pavement response. In terms of subgrade stiffness, both the vertical stiffness and shear resistance (how elastic or how liquid the subgrade is) need to be investigated.

5) More research on how the in-plane response affects opening and closing and how this affects joint stiffness, both linear and rotational stiffness. The affects of dowels on all of the joint properties above needs to be better defined.

6) In the FEACONS program, more efficient programming techniques and graphical presentation of outputs are strongly recommended.

7) In the INPLANE program, the assembly of the structural stiffness matrix should be modified for symmetric loadings and the planar results of concrete shrinkage and thermal effects should be combined with those of FEACONS program.

APPENDIX A

MAXIMUM STRESS TABLES

Notations : Numbers in parentheses denote node numbers at which maximum stress occurs and also alphabets in parentheses show approximate locations as follows :

- L means loading point.
- N means near the loading point (a quarter of slab length)
- C means between the two wheel load
- F means far away from the load point

TABLE A.1 MAXIMUM STRESS fx
DUE TO EJE 40 kIPS AXLE LOADING.

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-306.8 (326N)	-252.5 (326N)	-209.5 (313N)			
		200	-260.1 (339N)		-177.0 (236L)			
		750	-259.8 (105N)	-215.5 (131N)	181.4 (236L)			451.4 (475F)
	High 10000	200	-250.5 (339N)	256.9 (236L)	320.9 (236L)			
		750	-247.4 (339N)		324.2 (236L)			
		1500			325.0 (236L)		561.9 (236L)	668.7 (203L)
0.3 (M)	Low 1000	10	-296.3 (326N)	-242.7 (313N)	-196.3 (300N)	179.0 (203L)		
		200	-247.6 (326N)		159.4 (236L)			
		750	-243.4 (339N)	-195.4 (131N)	164.4 (236L)	197.3 (236L)		447.4 (475F)
	High 10000	200	-243.1 (339N)	225.1 (235L)	281.3 (235L)	377.0 (236L)		
		750	-240.1 (339N)		283.6 (235L)			
		1500			283.7 (235L)	381.6 (236L)	517.7 (203L)	656.7 (203L)
1.2 (H)	Low 1000	10	-280.8 (313N)	-230.0 (300N)	-176.1 (300N)	183.1 (475F)		
		200	-234.7 (98N)		135.2 (235N)			
		750	-234.0 (98N)	-168.9 (144N)	141.1 (236N)	183.4 (475F)		413.8 (472F)
	High 10000	200	-234.7 (111N)	178.7 (235L)	227.1 (235N)	310.4 (235L)		
		750	-233.5 (111N)		230.9 (235N)			
		1500			231.4 (235N)	315.7 (236L)	477.2 (203)	615.3 (203L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE A.2 MAXIMUM STRESS fy
DUE TO EJE 40 kIPS AXLE LOADING.

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	360.5 (216L)	417.6 (216L)	474.6 (216L)			
		200	207.4 (216L)		323.7 (216L)			
		750	171.5 (216L)	229.9 (216L)	288.0 (216L)			574.2 (216L)
	High 10000	200	209.3 (216L)	268.9 (216L)	328.6 (216L)			
		750	174.4 (203L)		293.2 (203L)			
		1500			286 (203L)		515 (203L)	593.5 (203L)
0.3 (M)	Low 1000	10	250.2 (216L)	321.3 (216L)	393.4 (216L)	510.4 (216L)		
		200	-207.9 (227C)		254.9 (216L)			
		750	-209.0 (227C)	145.7 (216L)	219.4 (216L)	338.5 (216L)		563.4 (216L)
	High 10000	200	-210.8 (227C)	186.8 (216L)	265.7 (216L)	394.7 (216L)		
		750	-211.2 (227C)		231.3 (203L)			
		1500			224.0 (203L)	353.0 (203L)	521.2 (203L)	605.2 (203L)
1.2 (H)	Low 1000	10	-264.2 (227C)	241.8 (216L)	320.7 (216L)	460.3 (216L)		
		200	-275.4 (227C)		213.0 (216L)			
		750	-276.9 (227C)	-202.0 (227C)	178.3 (216L)	318.1 (216L)		582.8 (216L)
	High 10000	200	-282.5 (227C)	-197.0 (227C)	221.5 (216L)	377.1 (216L)		
		750	-283.4 (227C)		188.6 (203L)			
		1500			181.5 (203L)	337.4 (203L)		643.6 (203L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE A.3 MAXIMUM STRESS fxy
DUE TO EJE 40 kIPS AXLE LOADING.

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-121.4 (250L)	116.0 (191L)	128.9 (191L)			
		200	-92.1 (232N)		98.5 (178N)			
		750	-97.4 (232N)	-96.3 (232N)	95.2 (232N)			170.4 (130N)
	High 10000	200	-63.0 (232N)	65.2 (178N)	69.2 (178L)			
		750	-68.30 (232N)		-68.6 (232N)			
		1500			-68.9 (232N)		94.1 (507F)	138.6 (507F)
0.3 (M)	Low 1000	10	-134.3 (250L)	-122.6 (250L)	111.1 (191L)	130.5 (191L)		
		200	-76.8 (232N)		82.2 (191L)			
		750	-82.2 (232N)	-80.2 (232N)	-78.5 (232N)	93.7 (178N)		151.6 (191L)
	High 10000	200	-69.0 (250L)	-63.8 (250L)	62.2 (178L)	68.1 (178N)		
		750	-61.8 (250L)		-58.3 (232N)			
		1500			-58.7 (232N)	61.5 (178N)	92.1 (507F)	147.2 (507F)
1.2 (H)	Low 1000	10	-137.5 (225C)	-124.9 (250L)	-113.4 (250L)	110.4 (191L)		
		200	-83.5 (250L)		64.3 (191L)			
		750	-75.4 (250L)	66.2 (211N)	68.9 (198N)	76.1 (191L)		164.2 (507F)
	High 10000	200	-74.6 (250L)	-69.2 (250L)	-62.4 (250L)	60.3 (178N)		
		750	-66.4 (250L)		-54.3 (250L)			
		1500			55.2 (198N)	57.2 (198N)	193.4 (235L)	163.2 (507F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE A.4 MAXIMUM PRINCIPAL STRESS f1
DUE TO EJE 40 kIPS AXLE LOADING.

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	274.2 (216L)	434.3 (216L)	495.5 (216L)			
		200	219.4 (216L)		344.0 (216L)			
		750	183.4 (216L)	246.2 (216L)	308.7 (216L)			621.1 (216L)
	High 10000	200	211.6 (216L)	272.6 (216L)	334.2 (216L)			688.1 (203L)
		750	199.3 (235L)		324.2 (236L)			
		1500			325.0 (236L)		575.4 (203L)	683.4 (203L)
0.3 (M)	Low 1000	10	259.4 (216L)	333.9 (216L)	409.6 (216L)	450.2 (216L)		
		200	-167.4 (384F)		270.4 (216L)			
		750	-167.4 (384F)	161.9 (203L)	235.3 (216L)	358.7 (216L)		604.4 (216L)
	High 10000	200	170.2 (235L)	225.2 (235L)	281.5 (235L)	401.7 (216L)		
		750	172.3 (235L)		284.0 (235L)			
		1500			284.1 (235L)	381.6 (236L)	548.4 (203L)	674.6 (203L)
1.2 (H)	Low 1000	10	-206.8 (488F)	249.1 (216L)	330.8 (216L)	475.6 (216L)		
		200	-206.8 (488F)		221.3 (216L)			
		750	-206.8 (488F)	133.8 (236L)	186.4 (216L)	330.4 (216L)		615.7 (216L)
	High 10000	200	-206.8 (488F)	178.7 (235L)	227.3 (235L)	381.7 (216L)		
		750	-206.8 (488F)		231.3 (235L)			
		1500			231.7 (235L)	341.3 (203L)	546.1 (203L)	664.2 (203L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE A.5 MAXIMUM PRINCIPAL STRESS f2
DUE TO EJE 40 kIPS AXLE LOADING.

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-268.1 (326N)	-252.5 (326N)	-211.1 (222L)			
		200	-261.2 (339N)		-171.6 (144N)			
		750	-260.7 (105N)	-215.5 (131N)	-176.3 (144N)			-421.5 (222L)
	High 10000	200	-251.1 (339N)	206.8 (203L)	274.5 (203L)			597.7 (203L)
		750	-248.2 (339N)		273.5 (203L)			
		1500			271.1 (203L)		499.1 (203L)	578.7 (203L)
0.3 (M)	Low 1000	10	-299.0 (326N)	-247.1 (213L)	-201.5 (300N)	-210.4 (222L)		
		200	-248.3 (326N)		152.0 (236L)			
		750	-243.5 (326N)	-196.1 (131N)	-155.5 (144N)	174.4 (203L)		-358.1 (222L)
	High 10000	200	-243.1 (339N)	-169.7 (326N)	226.5 (203L)	336.7 (203L)		
		750	-240.1 (339N)		222.8 (203L)			
		1500			218.4 (203L)	334.7 (203L)	490.5 (203L)	587.3 (203L)
1.2 (H)	Low 1000	10	-302.7 (227L)	-240.4 (300N)	-184.0 (300N)	168.7 (475F)		
		200	-279.3 (227L)		-124.9 (227L)			
		750	-279.4 (227L)	-202.6 (227L)	-130.3 (157N)	168.7 (475F)		320.8 (98F)
	High 10000	200	-284.2 (227L)	-197.7 (227L)	176.9 (203L)	285.3 (203L)		
		750	-284.1 (227L)		180.1 (203L)			
		1500			178.2 (203L)	290.9 (203L)	477.2 (203L)	594.8 (203L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE A.6 MAXIMUM PRINCIPAL STRESS Tmax
DUE TO EJE 40 kIPS AXLE LOADING.

Sub Ks	Joint Stiffness Kr	Joint Stiffness Kl	dT -10	dT -5	dT 0	dT +8	dT +18	dT +25
0.1 (S)	Low 1000	10	129.3 (216L)	191.0 (216L)	210.7 (216L)			
0.1 (S)	Low 1000	200	114.9 (118N)		137.4 (216L)			
0.1 (S)	Low 1000	750	147.4 (118N)	128.6 (118N)	120.5 (216L)			271.1 (222L)
0.1 (S)	High 10000	200	132.8 (118N)	108.1 (235L)	138.1 (235L)			306.9 (222L)
0.1 (S)	High 10000	750	135.4 (118N)		138.8 (235L)			
0.1 (S)	High 10000	1500			271.4 (235L)		249.1 (235L)	302.5 (222L)
0.3 (M)	Low 1000	10	151.3 (313N)	148.4 (216L)	176.4 (216L)	186.8 (216L)		
0.3 (M)	Low 1000	200	130.5 (118N)		109.1 (216L)			
0.3 (M)	Low 1000	750	133.4 (118N)	111.11 (144N)	97.6 (157N)	138.0 (216L)		234.2 (222L)
0.3 (M)	High 10000	200	125.6 (118N)	95.7 (227C)	119.3 (235L)	165.1 (235L)		
0.3 (M)	High 10000	750	128.6 (118N)		120.2 (235L)			
0.3 (M)	High 10000	1500			120.3 (235L)	166.3 (235L)	227.5 (235L)	282.7 (222L)
1.2 (H)	Low 1000	10	160.7 (226C)	136.5 (226N)	145.4 (216L)	204.1 (216L)		
1.2 (H)	Low 1000	200	133.1 (227C)		92.1 (216L)			
1.2 (H)	Low 1000	750	133.0 (227C)	106.2 (213)	81.0 (170N)	133.5 (216L)		236.4 (216L)
1.2 (H)	High 10000	200	114.9 (313N)	97.6 (227C)	89.3 (235L)	131.6 (235L)		
1.2 (H)	High 10000	750	116.5 (131N)		91.0 (235L)			
1.2 (H)	High 10000	1500			91.2 (235L)	133.8 (235L)	193.4 (235L)	237.0 (222L)

Temperature differentials dT(°F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9", Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE B.1 MAXIMUM STRESS fx
DUE TO EJC 40 KIPS AXLE LOADING.

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-247.5 (87F)	-175.6 (101N)	155.1 (206L)			
		200	-245.1 (85F)		162.9 (206L)			
		750	-245.1 (85F)	-169.2 (347N)	162.7 (238L)			451.4 (475F)
	High 10000	200	-239.7 (85F)	205.2 (206L)	273.9 (206L)			642 (205L)
		750	-239.7 (85F)		274.0 (206L)			
		1500			274.8 (205L)		538.8 (205L)	646.5 (205L)
0.3 (M)	Low 1000	10	-246.6 (100N)	-170.3 (114N)	129.3 (238L)	168.4 (206L)		
		200	-239.3 (98N)		139.8 (206L)			
		750	-239.3 (98N)	-157.8 (334N)	140.7 (238L)	179.3 (238L)		447.4 (475F)
	High 10000	200	-238.2 (98N)	160.0 (206L)	227.3 (206L)	334.3 (206L)		
		750	-238.4 (98N)		227.7 (206L)			
		1500			228.1 (238L)	334.8 (206L)	493.6 (205L)	627.8 (205L)
1.2 (H)	Low 1000	10	-242.4 (114N)	-165.1 (317N)	98.7 (238L)	183.1 (475F)		
		200	-231.3 (99N)		115.5 (238L)			
		750	-231.1 (99N)	-141.7 (123N)	117.5 (238L)	183.4 (475F)		513.8 (472F)
	High 10000	200	-230.4 (112N)	-137.2 (331N)	172.1 (206L)	274.5 (206L)		
		750	-229.9 (112N)		174.0 (238L)			
		1500			174.7 (238L)	276.0 (206L)	459.8 (205L)	580.1 (205L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE B.2 MAXIMUM STRESS fy
DUE TO EJC 40 KIPS AXLE LOADING.

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	364.3 (218L)	409.8 (218L)	455.1 (218L)			
		200	216.2 (218L)		311.3 (218L)			
		750	182.5 (218L)	230.3 (218L)	278.0 (218L)			513.1 (218L)
	High 10000	200	213.5 (218L)	262.8 (218L)	312.0 (218L)			573.7 (218L)
		750	183.3 (238L)		278.1 (218L)			
		1500			265.4 (218L)		455.9 (218L)	527.3 (218L)
0.3 (M)	Low 1000	10	264.5 (218L)	319.8 (218L)	376.6 (218L)	468.0 (218L)		
		200	-198.1 (215C)		242.1 (218L)			
		750	-198.0 (215C)	156.0 (238)	209.3 (218L)	305.7 (218L)		485.2 (218L)
	High 10000	200	-194.1 (215C)	184.2 (218L)	248.7 (218L)	353.6 (218L)		
		750	-193.4 (215C)		215.4 (218L)			
		1500			202.8 (238L)	305.3 (218L)	450.0 (218L)	525.6 (218L)
1.2 (H)	Low 1000	10	-260.0 (215C)	236.8 (218L)	300.9 (218L)	411.0 (218L)		
		200	-271.2 (215C)		192.4 (218L)			
		750	-271.7 (215C)	-183.3 (215C)	160.8 (218L)	274.2 (218L)		484.6 (218L)
	High 10000	200	-273.3 (215C)	-176.3 (215C)	198.8 (218L)	327.0 (218L)		
		750	-273.1 (215C)		167.5 (238L)			
		1500			162.0 (238L)	282.6 (218L)	453.3 (218L)	560.5 (218L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE B.3 MAXIMUM STRESS fxy
DUE TO EJC 40 KIPS AXLE LOADING.

Sub	Joint Stiffness		Temperature differentials dT(°F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	81.1 (250L)	94.7 (250L)	108.2 (275N)			
		200	-59.9 (234N)		88.0 (263N)			
		750	-66.5 (234N)	-69.8 (234N)	83.0 (288N)			183.9 (130N)
	High 10000	200	51.9 (263N)	56.5 (263N)	61.1 (263N)			138.6 (507F)
		750	-52.0 (247N)		56.3 (301N)			
		1500			-55.9 (247N)		94.1 (507F)	138.6 (507F)
0.3 (M)	Low 1000	10	-94.7 (216C)	-80.5 (216C)	89.5 (250L)	108.9 (224L)		
		200	-45.4 (210N)		70.1 (263N)			
		750	-54.1 (210N)	-53.7 (210N)	64.8 (263N)	84.4 (263N)		164.1 (143N)
	High 10000	200	44.1 (263N)	48.8 (263N)	53.4 (263N)	60.5 (263N)		
		750	-43.6 (210N)		48.1 (263N)			
		1500			46.6 (263N)	53.7 (263N)	92.1 (507F)	147.2 (507F)
1.2 (H)	Low 1000	10	-102.3 (216C)	-88.1 (216C)	-72.3 (216C)	90.2 (250L)		
		200	-52.3 (191L)		53.4 (263N)			
		750	-47.4 (191L)	-43.7 (210N)	48.3 (263N)	65.7 (263N)		164.2 (507F)
	High 10000	200	-47.8 (191L)	-42.0 (191L)	45.6 (263N)	52.8 (263N)		
		750	-42.6 (191L)		40.5 (263N)			
		1500			39.0 (263N)	46.2 (263N)	99.4 (505F)	163.2 (507F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE B.4 MAXIMUM PRINCIPAL STRESS f1
DUE TO EJC 40 KIPS AXLE LOADING.

Sub Ks	Joint Stiffness Kr	Joint Stiffness Kl	Temperature differentials dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	364.7 (218L)	411.9 (218L)	460.0 (218L)			
		200	217.8 (218L)		321.2 (218L)			
		750	190.3 (238L)	235.6 (218L)	288.5 (218L)			564.1 (218L)
	High 10000	200	213.7 (218L)	263.6 (218L)	314.4 (218L)			656.4 (206L)
		750	188.0 (238L)		291.4 (238L)			
		1500			287.8 (238L)		542.7 (205L)	652.2 (205L)
0.3 (M)	Low 1000	10	264.8 (218L)	320.0 (218L)	378.3 (218L)	474.1 (218L)		
		200	-169.2 (475F)		247.2 (218L)			
		750	-169.1 (475F)	169.8 (238L)	220.5 (238L)	318.7 (218L)		531.6 (218L)
	High 10000	200	-169.1 (475F)	192.4 (238L)	252.3 (238L)	358.4 (218L)		
		750	-169.1 (475F)		240.2 (238L)			
		1500			237.3 (238L)	345.0 (206L)	500.4 (206L)	634.0 (205L)
1.2 (H)	Low 1000	10	-206.8 (488F)	237.1 (218L)	301.1 (218L)	414.4 (218L)		
		200	-210.7 (111N)		194.0 (218L)			
		750	-211.5 (124N)	-121.7 (306N)	172.5 (238L)	281.9 (218L)		528.6 (218L)
	High 10000	200			199.8 (238L)			
		750			186.4 (238L)			
		1500			183.0 (238L)	290.0 (206L)	474.8 (205L)	598.1 (205L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE B.6 . MAXIMUM PRINCIPAL STRESS Tmax
DUE TO EJC 40 KIPS AXLE LOADING.

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	168.5 (218L)	179.7 (218L)	191.9 (218L)			
		200	121.6 (118N)		123.8 (218L)			
		750	122.5 (118N)	108.2 (118N)	108.2 (218L)			244.9 (222L)
	High 10000	200	113.7 (118N)	84.9 (118N)	71.1 (209L)			254.7 (222L)
		750	114.5 (118N)		73.8 (209L)			
		1500			74.1 (209L)		200.6 (222L)	256.2 (222L)
0.3 (M)	Low 1000	10	127.3 (218L)	143.3 (218L)	161.3 (218L)	192.5 (218L)		
		200	102.7 (118N)		96.0 (218L)			
		750	103.4 (118N)	82.6 (118N)	80.3 (218L)	117.2 (218L)		210.7 (222L)
	High 10000	200	102.2 (118N)	79.7 (215C)	73.8 (215C)	101.1 (222L)		
		750	102.9 (118N)		74.0 (215C)			
		1500			73.9 (215C)	102.7 (222L)	180.4 (222L)	240.9 (222L)
1.2 (H)	Low 1000	10	138.3 (215L)	110.6 (218L)	133.3 (218L)	174.9 (218L)		
		200	130.3 (215L)		78.7 (218L)			
		750	130.6 (215L)	95.0 (215C)	63.4 (218L)	109.7 (218L)		223.7 (53F)
	High 10000	200			68.3 (215C)			
		750			68.8 (215C)			
		1500			68.8 (215C)	77.1 (222L)	156.7 (482F)	214 (53F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE C.1 MAXIMUM STRESS fx
DUE TO EME 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	241.3 (261L)	327.4 (261L)	413.4 (261L)			
		200	240.9 (261L)		413.2 (261L)			
		750	240.9 (261L)	327.0 (261L)	413.1 (261L)			860.3 (262L)
	High 10000	200	249.0 (261L)	331.6 (261L)	414.2 (261L)			847.7 (262L)
		750	248.9 (261L)		414.1 (261L)			
		1500			414.1 (261L)		724.4 (262L)	847.6 (262L)
0.3 (M)	Low 1000	10	-233.8 (157N)	289.2 (261L)	375.9 (261L)	516.4 (261L)		
		200	-234.7 (157N)		375.9 (261L)			
		750	-234.7 (157N)	289.3 (261L)	375.9 (261L)	516.4 (261L)		843.6 (242L)
	High 10000	200	-233.9 (365N)	293.6 (261L)	376.6 (261L)	511.3 (261L)		
		750	-234.2 (365)		376.6 (261L)			
		1500			376.6 (261L)	511.3 (261L)	687.2 (262L)	834.2 (242L)
1.2 (H)	Low 1000	10	-240.0 (345N)	230.1 (261L)	311.7 (261L)	447.7 (261L)		
		200	-240.4 (345N)		311.7 (261L)			
		750	-240.4 (345N)	230.3 (261L)	311.7 (261L)	447.5 (261L)		853.5 (242)
	High 10000	200	-238.9 (150N)	231.8 (261L)	311.8 (261L)	443.6 (261L)		
		750	-239.0 (150N)		311.8 (261L)			
		1500			311.8 (261L)	443.6 (261L)	647.1 (242)	821.3 (242)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9", Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE C.2 MAXIMUM STRESS fy
DUE TO EME 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	167.0 (242L)	223.9 (242L)	280.9 (242L)			
		200	166.1 (242L)		280.2 (242L)			
		750	166.0 (242L)	223.0 (242L)	280.1 (242L)			576.5 (242)
	High 10000	200	161.0 (242L)	220.3 (242L)	279.7 (242L)			601.9 (242)
		750	160.9 (242L)		279.6 (242L)			
		1500			279.5 (242L)		508.1 (242)	601.6 (242)
0.3 (M)	Low 1000	10	-189.7 (240C)	139.1 (242L)	220.6 (242L)	368.2 (242L)		
		200	-190.0 (240C)		220.5 (242L)			
		750	-190.0 (240C)	139.0 (242L)	220.5 (242L)	367.8 (242L)		611.0 (242L)
	High 10000	200	-190.9 (240C)	138.6 (242L)	220.5 (242L)	368.5 (242L)		
		750	-191.0 (240C)		220.5 (242L)			
		1500			220.5 (242L)	368.4 (242L)	532.0 (242L)	615.4 (242L)
1.2 (H)	Low 1000	10	-266.9 (253C)	-176.7 (253C)	178.1 (242L)	364.6 (242L)		
		200	-266.8 (253C)		178.2 (242L)			
		750	-266.8 (253C)	-176.7 (253C)	178.2 (242L)	364.5 (242L)		663.7 (242L)
	High 10000	200	-266.0 (253C)	-176.5 (253C)	178.2 (242L)	361.7 (242L)		
		750	-266.0 (253C)		178.2 (242L)			
		1500			178.2 (242L)	361.7 (242L)	537.8 (242L)	673.2 (242L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9", Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE C.3 MAXIMUM STRESS fxy
DUE TO EME 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-59.5 (377N)	-50.7 (283N)	-49.9 (283N)			
		200	-60.5 (390F)		-49.9 (283N)			
		750	-61.0 (390F)	-50.7 (310N)	-49.9 (283N)			135.7 (507F)
	High 10000	200	-50.3 (283N)	-50.0 (283N)	-49.8 (283N)			132.7 (507F)
		750	-50.3 (283N)		-49.8 (283N)			
		1500			-49.8 (283N)		91.8 (507F)	132.7 (507F)
0.3 (M)	Low 1000	10	-49.1 (289N)	-49.2 (289N)	-49.5 (289N)	-48.9 (289N)		
		200	-49.1 (289N)		-49.5 (289N)			
		750	-49.1 (289N)	-49.2 (289N)	-49.4 (289N)	-48.8 (289N)		136.9 (507F)
	High 10000	200	-49.5 (289N)	-49.4 (289N)	-49.5 (289N)	-48.6 (289N)		
		750	-49.5 (289N)		-49.5 (289N)			
		1500			-49.5 (289N)	-48.5 (289N)	88.7 (507F)	136.2 (507F)
1.2 (H)	Low 1000	10	-50.2 (289N)	-49.6 (289N)	-49.1 (289N)	-45.9 (289N)		
		200	-50.5 (289N)		-49.1 (289N)			
		750	-50.5 (289N)	-49.6 (289N)	-49.1 (289N)	-45.5 (289N)		155.9 (507F)
	High 10000	200	-50.6 (289N)	-49.6 (289N)	-49.1 (289N)	-47.1 (289N)		
		750	-50.6 (289N)		-49.1 (289N)			
		1500			-49.1 (289N)	-47.1 (289N)	105.5 (507F)	155.2 (507F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9", Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE C.4 MAXIMUM PRINCIPAL STRESS f1
DUE TO EME 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	241.4 (261L)	327.4 (261L)	413.5 (261L)			
		200	241.0 (261L)		413.2 (261L)			
		750	240.9 (261L)	327.0 (261L)	413.1 (261L)			860.4 (262L)
	High 10000	200	249.0 (261L)	331.6 (261L)	414.2 (261L)			847.9 (262L)
		750	249.0 (261L)		414.1 (261L)			
		1500			414.1 (261L)		724.5 (262L)	847.8 (262L)
0.3 (M)	Low 1000	10	203.9 (261L)	289.2 (261L)	375.9 (261L)	516.5 (261L)		
		200	203.9 (261L)		375.9 (261L)			
		750	203.9 (261L)	289.3 (261L)	375.9 (261L)	516.4 (261L)		843.9 (242L)
	High 10000	200	211.4 (261L)	293.6 (261L)	376.6 (261L)	511.3 (261L)		
		750	211.4 (261L)		376.6 (261L)			
		1500			376.6 (261L)	511.3 (261L)	687.3 (262L)	834.5 (242L)
1.2 (H)	Low 1000	10	-207.0 (188N)	230.1 (261L)	311.7 (261L)	447.7 (261L)		
		200	-207.1 (188N)		311.7 (261L)			
		750	-207.1 (188N)	230.3 (261L)	311.7 (261L)	447.5 (261L)		854.2 (242L)
	High 10000	200	-205.6 (175N)	231.8 (261L)	311.8 (261L)	443.3 (261L)		
		750	-205.6 (175N)		311.8 (261L)			
		1500			311.8 (261L)	443.6 (261L)	647.7 (242L)	821.8 (242L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE C.5 MAXIMUM PRINCIPAL STRESS f2
DUE TO EME 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-223.0 (144N)	222.5 (242L)	280.1 (242L)			
		200	-226.0 (144N)		279.5 (242L)			
		750	-226.4 (144N)	221.7 (242L)	279.4 (242L)			576.3 (242L)
	High 10000	200	-235.0 (365N)	219.0 (242L)	278.9 (242L)			601.6 (242L)
		750	-236.0 (365N)		278.8 (242L)			
		1500			278.8 (242L)		507.8 (242L)	601.3 (242L)
0.3 (M)	Low 1000	10	-233.8 (157N)	-152.4 (157N)	219.9 (242L)	367.6 (242L)		
		200	-234.8 (157N)		219.8 (242L)			
		750	-234.9 (157N)	-153.6 (352N)	219.7 (242L)	367.1 (242L)		610.7 (242L)
	High 10000	200	-233.9 (365N)	-156.6 (365N)	219.8 (242L)	367.9 (242L)		
		750	-234.0 (365N)		219.8 (242L)			
		1500			219.8 (242L)	367.8 (242L)	531.5 (242L)	615.1 (242L)
1.2 (H)	Low 1000	10	-267.0 (253C)	-176.7 (253C)	177.2 (242)	363.7 (242L)		
		200	-266.8 (253C)		177.2 (242)			
		750	-266.8 (253C)	-176.7 (253C)	177.2 (242)	363.4 (242L)		663.1 (242L)
	High 10000	200	-266.0 (253C)	-176.5 (253C)	177.2 (242)	360.7 (242L)		
		750	-266.0 (253C)		177.2 (242)			
		1500			177.2 (242)	360.7 (242L)	537.2 (242L)	672.6 (242L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9", Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE C.6 MAXIMUM PRINCIPAL STRESS Tmax
DUE TO EME 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	125.1 (157N)	139.0 (261L)	180.7 (261L)			
		200	126.0 (157N)		180.5 (261L)			
		750	126.2 (157N)	138.8 (261L)	180.5 (261L)			392.0 (261L)
	High 10000	200	127.4 (157N)	141.1 (261L)	181.0 (261L)			38.3 (261L)
		750	127.4 (157N)		181.0 (261L)			
		1500			181.0 (261L)		326.2 (261L)	383.2 (261L)
0.3 (M)	Low 1000	10	126.0 (157N)	119.3 (261L)	161.3 (261L)	229.5 (261L)		
		200	126.5 (157N)		161.3 (261L)			
		750	126.5 (157N)	119.3 (261L)	161.3 (261L)	229.5 (261L)		380.5 (261L)
	High 10000	200	126.6 (157N)	121.5 (261L)	161.7 (261L)	226.9 (261L)		
		750	126.6 (157N)		161.7 (261L)			
		1500			161.7 (261L)	226.9 (261L)	309.0 (261L)	369.1 (261L)
1.2 (H)	Low 1000	10	121.9 (170N)	89.0 (261L)	128.4 (261L)	194.3 (261L)		
		200	122.1 (170N)		128.4 (261L)			
		750	122.1 (170N)	89.1 (261L)	128.4 (261L)	194.3 (261L)		351.2 (261L)
	High 10000	200	120.5 (170N)	89.9 (261L)	128.5 (261L)	192.3 (261L)		
		750	120.5 (170N)		128.5 (261L)			
		1500			128.5 (261L)	192.2 (261L)	278.9 (261L)	337.1 (261L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE D.1 MAXIMUM STRESS fx
DUE TO EMC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-189.3 (462F)	254.6 (257L)	346.3 (257L)			
		200	-189.6 (157N)		346.2 (257L)			
		750	-189.8 (157N)	254.3 (257L)	346.2 (257L)			820.2 (257L)
	High 10000	200	-210.7 (136N)	255.6 (257L)	346.4 (257L)			820.3 (257L)
		750	-210.7 (136N)		346.3 (257L)			
		1500			346.3 (257L)		683.3 (257L)	820.3 (257L)
0.3 (M)	Low 1000	10	-217.8 (161N)	190.4 (257L)	289.2 (257L)	456.1 (257L)		
		200	-219.0 (150N)		289.2 (257L)			
		750	-219.1 (150N)	190.3 (257L)	289.2 (257L)	456.4 (257L)		814.8 (257L)
	High 10000	200	-227.9 (150N)	194.0 (257L)	289.7 (257L)	451.8 (257L)		
		750	-227.9 (150N)		289.7 (257L)			
		1500			289.7 (257L)	451.8 (257L)	652.2 (257L)	802.7 (257L)
1.2 (H)	Low 1000	10	-237.0 (163N)	-135.8 (162N)	223.9 (277L)	401.9 (257L)		
		200	-237.6 (163N)		223.9 (277L)			
		750	-237.6 (163N)	-135.9 (162N)	223.9 (277L)	402.9 (257L)		812.6 (257L)
	High 10000	200	-235.6 (163N)	-134.3 (162N)	224.0 (277L)	392.4 (257L)		
		750	-235.7 (163N)		224.0 (277L)			
		1500			224.0 (277L)	392.4 (257L)	617.8 (257L)	778.4 (257L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE D.2 MAXIMUM STRESS fy
DUE TO EMC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	175.8 (277L)	211.7 (277L)	254.2 (257L)			
		200	175.1 (277L)		253.5 (257L)			
		750	175.0 (277L)	210.9 (277L)	253.4 (257L)			507.4 (257L)
	High 10000	200	171.4 (277L)	208.9 (277L)	253.0 (257L)			523.7 (257L)
		750	171.3 (277L)		252.9 (257L)			
		1500			252.9 (257L)		446.1 (257L)	523.5 (257L)
0.3 (M)	Low 1000	10	-186.7 (280C)	149.1 (277L)	198.9 (277L)	314.1 (257L)		
		200	-187.0 (280C)		198.8 (277L)			
		750	-187.0 (280C)	148.9 (277L)	198.7 (277L)	313.7 (257L)		513.6 (257L)
	High 10000	200	-188.2 (280C)	148.6 (277L)	198.8 (277L)	314.3 (257L)		
		750	-188.2 (280C)		198.7 (277L)			
		1500			198.7 (277L)	314.3 (257L)	456.8 (257L)	531.6 (257L)
1.2 (H)	Low 1000	10	-271.2 (280C)	-168.2 (280C)	158.4 (277L)	302.6 (257L)		
		200	-271.0 (280C)		158.4 (277L)			
		750	-271.0 (280C)	-168.2 (280C)	158.4 (277L)	302.4 (257L)		578.9 (257L)
	High 10000	200	-269.9 (280C)	-167.8 (280C)	158.4 (277L)	297.0 (257L)		
		750	-269.9 (280C)		158.4 (277L)			
		1500			158.4 (277L)	297.0 (257L)	467.7 (257L)	578.6 (257L)

Temperature differentials dT(^{o}F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE D.3 MAXIMUM STRESS fxy
DUE TO EMC 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT (^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-63.5 (390F)	-51.5 (377N)	-43.6 (223N)			
		200	-67.0 (390F)		-43.7 (223N)			
		750	-67.7 (390F)	-52.9 (377N)	-43.7 (223N)			135.6 (507F)
	High 10000	200	-48.8 (507F)	-43.9 (223N)	-43.4 (223N)			132.7 (507F)
		750	-48.8 (507F)		-43.4 (223N)			
		1500			-43.3 (223N)		91.8 (507F)	132.6 (507F)
0.3 (M)	Low 1000	10	-38.5 (505F)	-36.4 (250N)	-36.5 (250N)	37.8 (250N)		
		200	-39.9 (414F)		-36.5 (250N)			
		750	-40.3 (414F)	-36.4 (250N)	-36.5 (250N)	37.7 (250N)		136.8 (507F)
	High 10000	200	-38.5 (505F)	-36.5 (250N)	-36.5 (250N)	-37.7 (250N)		
		750	-38.5 (505F)		-36.5 (250N)			
		1500			-36.5 (250N)	-37.7 (250N)	88.7 (507F)	136.1 (507F)
1.2 (H)	Low 1000	10	-34.8 (504F)	-30.8 (250N)	-31.6 (250N)	-35.2 (250N)		
		200	-34.8 (504F)		-31.6 (250N)			
		750	-34.8 (504F)	-30.9 (250N)	-31.6 (250N)	-35.3 (250N)		155.9 (507F)
	High 10000	200	-34.8 (504F)	-31.0 (250N)	-31.6 (250N)	-34.1 (250N)		
		750	-34.8 (504F)		-31.6 (250N)			
		1500			-31.6 (250N)	-34.1 (250N)	105.5 (507F)	155.1 (507F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE D.4 MAXIMUM PRINCIPAL STRESS f1
DUE TO EMC 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	179.4 (277L)	256.1 (257L)	347.0 (257L)			
		200	178.6 (277L)		346.9 (257L)			
		750	178.4 (277L)	255.8 (257L)	346.9 (257L)			820.3 (257L)
	High 10000	200	176.3 (277L)	256.9 (257L)	347.1 (257L)			820.5 (257L)
		750	176.2 (277L)		347.1 (257L)			
		1500			347.1 (257L)		683.5 (257L)	820.5 (257L)
0.3 (M)	Low 1000	10	-168.1 (46 F)	191.8 (277L)	289.9 (257L)	456.7 (257L)		
		200	-168.0 (46 F)		289.9 (257L)			
		750	-168.0 (46 F)	191.6 (277L)	289.9 (257L)	456.9 (257L)		815.0 (257L)
	High 10000	200	-168.2 (46 F)	195.1 (277L)	290.5 (257L)	452.3 (257L)		
		750	-168.2 (46 F)		290.5 (257L)			
		1500			290.5 (257L)	452.3 (257L)	652.5 (257L)	802.9 (257L)
1.2 (H)	Low 1000	10	-222.2 (345N)	131.1 (277L)	225.0 (277L)	402.8 (257L)		
		200	-222.6 (345N)		225.0 (277L)			
		750	-222.7 (345N)	131.2 (277L)	225.0 (277L)	403.8 (257L)		813.0 (257L)
	High 10000	200	-221.7 (345N)	132.9 (277L)	225.1 (277L)	393.2 (257L)		
		750	-221.7 (345N)		225.1 (277L)			
		1500			225.1 (277L)	393.2 (257L)	618.3 (257L)	778.7 (257L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE D.5 MAXIMUM PRINCIPAL STRESS f2
DUE TO EMC 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(°F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-201.7 (157N)	209.3 (277L)	253.5 (257L)			
		200	-203.4 (157N)		252.8 (257L)			
		750	-203.7 (157N)	208.7 (277L)	252.7 (257L)			507.3 (257L)
	High 10000	200	-213.6 (135N)	206.8 (277L)	252.3 (257L)			523.5 (257L)
		750	-213.6 (135N)		252.2 (257L)			
		1500			252.2 (257L)		445.9 (257L)	523.3 (257L)
0.3 (M)	Low 1000	10	-218.9 (161N)	147.3 (277L)	198.0 (277L)	313.5 (257)		
		200	-219.6 (161N)		197.8 (277L)			
		750	-219.6 (161N)	147.1 (277L)	197.8 (277L)	313.2 (257L)		531.4 (257L)
	High 10000	200	-227.9 (150N)	146.9 (277L)	197.8 (277L)	313.8 (257L)		
		750	-227.9 (150N)		197.8 (277L)			
		1500			197.8 (277L)	313.8 (257L)	456.4 (257L)	531.4 (257L)
1.2 (H)	Low 1000	10	-271.2 (280C)	-168.2 (280C)	157.8 (277L)	301.7 (257L)		
		200	-271.0 (280C)		157.3 (277L)			
		750	-271.0 (280C)	-168.2 (280C)	157.3 (277L)	301.5 (257L)		578.5 (257L)
	High 10000	200	-269.9 (280C)	-167.8 (280C)	157.3 (277L)	296.2 (257L)		
		750	-269.9 (280C)		157.3 (277L)			
		1500			157.3 (277L)	296.2 (257L)	467.3 (257L)	518.3 (257L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE D.6 MAXIMUM PRINCIPAL STRESS Tmax
DUE TO EMC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Joint Stiffness Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	115.8 (157N)	80.5 (157N)	75.7 (267C)			
		200	116.6 (157N)		76.1 (267C)			
		750	116.7 (157N)	81.3 (157N)	76.1 (267C)			276.0 (287N)
	High 10000	200	114.2 (157N)	79.0 (157N)	76.4 (267C)			268.5 (287N)
		750	114.3 (157N)		76.4 (267C)			
		1500			76.4 (267C)		211.5 (300N)	268.4 (287N)
0.3 (M)	Low 1000	10	106.0 (157N)	70.5 (267C)	79.4 (267C)	120.1 (234N)		
		200	106.3 (157N)		79.4 (267C)			
		750	106.4 (157N)	70.5 (267C)	79.4 (267C)	120.1 (234N)		268.4 (287N)
	High 10000	200	105.0 (157N)	72.5 (267C)	79.7 (267C)	117.7 (234N)		
		750	105.0 (157N)		79.0 (267C)			
		1500			79.7 (267C)	117.7 (234N)	199.4 (234N)	258.8 (234N)
1.2 (H)	Low 1000	10	97.7 (183N)	68.4 (267C)	67.6 (267C)	99.8 (288N)		
		200	97.8 (183N)		67.6 (267C)			
		750	97.8 (183N)	68.5 (267C)	67.6 (267C)	100.0 (288N)		256.1 (300N)
	High 10000	200	95.9 (170N)	69.2 (267C)	67.7 (267C)	97.4 (288N)		
		750	95.9 (170N)		67.7 (267C)			
		1500			67.7 (267C)	97.4 (288N)	181.8 (234N)	241.8 (234N)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE E.1 MAXIMUM STRESS fx
DUE TO TJE 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{O}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-226.6 (79 F)	172.4 (287L)	200.7 (287L)			
		200	-229.2 (352F)		189.5 (287L)			
		750	-229.6 (352F)	163.6 (170L)	188.0 (287L)			451.4 (475L)
	High 10000	200	-232.1 (352F)	162.3 (170L)	225.1 (287L)			566.0 (150L)
		750	-232.4 (352F)		225.4 (170L)			
		1500			225.7 (170L)		454.7 (287L)	566.0 (150L)
0.3 (M)	Low 1000	10	-227.9 (72 F)	171.8 (287L)	203.5 (287L)	253.3 (287L)		
		200	-228.1 (72 F)		193.1 (287L)			
		750	-228.1 (72 F)	167.2 (170L)	191.6 (287L)	241.9 (287L)		447.4 (475L)
	High 10000	200	-227.7 (85 F)	148.9 (170L)	210.1 (287L)	310.8 (287L)		
		750	-227.7 (85 F)		210.5 (170L)			
		1500			210.8 (170L)	309.0 (287L)	437.0 (287L)	566.2 (151L)
1.2 (H)	Low 1000	10	-227.5 (488F)	155.6 (287L)	197.8 (287L)	262.0 (287L)		
		200	-227.5 (488F)		187.0 (170L)			
		750	-227.5 (488F)	161.1 (170L)	188.4 (170L)	249.2 (287L)		513.8 (472L)
	High 10000	200	-227.7 (488F)	-136.8 (339N)	187.2 (287L)	289.7L (287)		
		750	-227.7 (488F)		188.4 (170L)			
		1500			188.7 (170L)	287.5 (287L)	430.6 (151L)	555.5 (151L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE E.2 MAXIMUM STRESS fy
DUE TO TJE 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-120 (8 F)	123.6 (268L)	181.9 (268L)			
		200	-120 (8 F)		174.1 (268L)			
		750	-120 (8 F)	114.6 (268L)	172.4 (268L)			455.5 (268L)
	High 10000	200	-123.2 (436F)	115.0 (268L)	174.7 (268L)			482.0 (268L)
		750	-122.8 (436F)		173.0 (268L)			
		1500			172.5 (268L)		400.2 (268L)	480.0 (268L)
0.3 (M)	Low 1000	10	-174.3 (149C)	-106.7 (149C)	131.4 (268L)	253.5 (268L)		
		200	-169.4 (149C)		124.2 (268L)			
		750	-169.2 (475F)	-102.1 (149C)	122.6 (268L)	243.9 (268L)		461.6 (268L)
	High 10000	200	-175.0 (149C)	-103.8 (149C)	126.2 (268L)	256.3 (268L)		
		750	-174.6 (149C)		124.6 (268L)			
		1500			124.1 (268L)	254.2 (268L)	421.0 (268L)	499.8 (268L)
1.2 (H)	Low 1000	10	-228.6 (149C)	-146.8 (149C)	102.3 (268L)	247.1 (268L)		
		200	-224.5 (149C)		97.3 (268L)			
		750	-224.1 (149C)	-143.0 (149C)	95.8 (268L)	246.8 (151L)		498.8 (268L)
	High 10000	200	-235.4 (266C)	-150.5 (266C)	97.9 (268L)	256.0 (268L)		
		750	-235.0 (266C)		96.5 (268L)			
		1500			96.0 (268L)	255.9 (151L)	444.9 (268L)	550.0 (268L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE E.3 MAXIMUM STRESS fxy
DUE TO TJE 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	55.4 (145N)	49.5 (145N)	48.9 (114N)			
		200	-53 (301L)		48.8 (114N)			
		750	-54.2 (301L)	-45.6 (295N)	48.9 (114N)			141.9 (507)
	High 10000	200	-49.9 (507F)	-44.1 (295N)	43.8 (114N)			138.6 (507F)
		750	-49.9 (507F)		43.8 (114N)			
		1500			43.9 (114N)		94.1 (507F)	138.6 (507F)
0.3 (M)	Low 1000	10	59.6 (145N)	56.2 (145N)	52.6 (145N)	54.7 (262N)		
		200	-61.4 (301L)		47.2 (145N)			
		750	-62.5 (301L)	-50.2 (301L)	46.2 (145N)	48.1 (262N)		148.2 (507F)
	High 10000	200	50.2 (145N)	49.1 (145N)	48.0 (145N)	46.6 (145N)		
		750	-50.7 (301L)		46.8 (145N)			
		1500			46.5 (145N)	45.3 (145N)	92.1 (507F)	147.2 (507F)
1.2 (H)	Low 1000	10	58.0 (131N)	55.7 (145N)	53.9 (145N)	52.8 (145N)		
		200	-61.4 (301L)		49.2 (145N)			
		750	-62.5 (301L)	-53.2 (301L)	48.1 (145N)	48.2 (145N)		164.2 (507F)
	High 10000	200	-53.5 (314N)	51.3 (145N)	49.3 (145N)	48.7 (145N)		
		750	-54.2 (314N)		48.3 (145N)			
		1500			48.0 (145N)	47.5 (145N)	99.4 (505F)	163.2 (507F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE E.4 MAXIMUM PRINCIPAL STRESS f1
DUE TO TJE 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Joint Stiffness Kl	Temperature differentials $dT(^{O}F)$ -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	146.5 (287L)	172.4 (287L)	202.3 (287L)			
		200	144.4 (170L)		190.3 (287L)			
		750	144.5 (170L)	164.5 (170L)	188.6 (287L)			479.7 (151L)
	High 10000	200	-121.2 (462F)	164.1 (170L)	225.1 (287L)			575.4 (151L)
		750	-121.1 (462F)		226.0 (170L)			
		1500			226.2 (170L)		456.4 (151L)	574.9 (151L)
0.3 (M)	Low 1000	10	-169.2 (475F)	172.9 (287L)	203.5 (287L)	255.9 (287L)		
		200	-169.2 (475F)		193.2 (287L)			
		750	-169.2 (475F)	170.1 (170L)	192.7 (170L)	251.1 (151L)		484.2 (151L)
	High 10000	200	-169.1 (475F)	152.6 (170L)	211.3 (170L)	310.8 (287L)		
		750	-169.1 (475F)		212.0 (170L)			
		1500			212.2 (170L)	309.0 (287L)	445.7 (151L)	576.0 (151L)
1.2 (H)	Low 1000	10	-206.8 (488F)	158.7 (170L)	198.0 (287L)	262.5 (287L)		
		200	-206.8 (488F)		189.4 (170L)			
		750	-206.8 (488F)	163.6 (170L)	190.2 (170L)	249.4 (287L)		513.8 (152L)
	High 10000	200	-206.8 (488F)	135.2 (170L)	189.3 (170L)	289.7 (287L)		
		750	-206.8 (488F)		190.3 (170L)			
		1500			190.5 (170L)	287.5 (287L)	453.5 (151L)	572.3 (151L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE E.5 MAXIMUM PRINCIPAL STRESS f2
DUE TO TJE 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-226.8 (79 F)	-158.0 (92 N)	115.2 (267L)			
		200	-229.3 (352F)		116.5 (150L)			
		750	-229.6 (352F)	-154.4 (352F)	116.8 (150L)			-347.5 (222N)
	High 10000	200	-232.1 (352F)	-153.7 (92 N)	150.0 (267L)			480.4 (268L)
		750	-232.4 (352F)		149.0 (267L)			
		1500			148.6 (267L)		398.3 (267L)	479.3 (267L)
0.3 (M)	Low 1000	10	-228.2 (72 F)	-147.6 (92 N)	101.5 (267L)	178.1 (151L)		
		200	-228.3 (72 F)		101.1 (151L)			
		750	-228.3 (72 F)	-145.2 (339N)	101.3 (151L)	178.0 (150L)		358.2 (150L)
	High 10000	200	-228.4 (85 F)	-146.7 (339N)	113.7 (151L)	241.6 (267L)		
		750	-228.4 (85 F)		114.2 (151L)			
		1500			114.3 (151L)	240.5 (267L)	405.8 (267L)	498.6 (268L)
1.2 (H)	Low 1000	10	-228.7 (149C)	-146.8 (149C)	90.3 (151L)	194.9 (151L)		
		200	-227.5 (488F)		93.1 (151L)			
		750	-227.5 (488F)	-144.4 (266C)	93.5 (151L)	191.2 (151L)		381.9 (150L)
	High 10000	200	-237.1 (266C)	-151.3 (266C)	93.7 (151L)	234.3 (151L)		
		750	-236.7 (266C)		94.2 (151L)			
		1500			94.4 (151L)	234.0 (151L)	426.1 (150L)	538.3 (150L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE E.6 MAXIMUM PRINCIPAL STRESS Tmax DUE TO TJE 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Joint Stiffness Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	117.0 (92 N)	84.4 (92 N)	83.5 (287L)			
		200	114.4 (92 N)		77.7 (287L)			
		750	114.1 (92 N)	81.6 (92 N)	76.9 (287L)			209.5 (222N)
	High 10000	200	119.3 (92 N)	82.0 (92 N)	93.0 (287L)			255.4 (287L)
		750	119.0 (92 N)		92.3 (287L)			
		1500			92.2 (287L)		207.7 (287L)	254.2 (287L)
0.3 (M)	Low 1000	10	113.3 (92 N)	78.8 (118N)	79.7 (287L)	107.7 (287L)		
		200	111.1 (92 N)		75.1 (287L)			
		750	110.7 (92 N)	75.2 (92 N)	74.5 (287L)	101.8 (287L)		207.6 (481F)
	High 10000	200	114.0 (92 N)	77.3 (105N)	83.2 (287L)	132.0 (287L)		
		750	113.7 (92 N)		82.7 (287L)			
		1500			82.6 (287L)	131.2 (287L)	195.3 (287L)	245.7 (287L)
1.2 (H)	Low 1000	10	109.1 (105N)	77.4 (118N)	73.6 (287L)	104.6 (287L)		
		200	110.2 (339N)		69.1 (287L)			
		750	110.6 (339N)	-73.3 (339N)	68.5 (287L)	98.4 (287L)		222.7 (469F)
	High 10000	200	108.5 (339N)	76.1 (118N)	69.2 (287L)	117.9 (287L)		
		750	108.8 (339N)		68.7 (287L)			
		1500			68.6 (287L)	116.9 (287L)	180.6 (287L)	227.8 (287L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi

Notations : S = Soft, M = Medium, H = Hard (Econocrete)

Ks = Spring stiffness of subgrade.

Kr = Rotational spring stiffness of Joint.

Kl = Linear spring stiffness of Joint.

dT = Temperature differential (Top - Bottom)

TABLE F.1 MAXIMUM STRESS fx
DUE TO TJC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-225.2 (72 F)	-134.7 (72 F)	129.5 (153L)			
		200	-224.8 (72 F)		125.4 (153L)			
		750	-224.8 (72 F)	-134.4 (73 F)	125.2 (153L)			451.4 (475F)
	High 10000	200	-227.6 (72 F)	-134.1 (72 F)	161.8 (153L)			557.7 (153L)
		750	-227.6 (72 F)		161.7 (153L)			
		1500			161.6 (153L)		442.5 (153L)	557.3 (153L)
0.3 (M)	Low 1000	10	-227.9 (72 F)	-129.7 (85 F)	116.7 (290L)	188.9 (153L)		
		200	-227.6 (72 F)		111.5 (290L)			
		750	-227.6 (72 F)	-129.4 (85 F)	111.1 (290L)	183.4 (153L)		447.4 (475F)
	High 10000	200	-227.4 (85 F)	-131.0 (85 F)	130.2 (153L)	252.7 (153L)		
		750	-227.4 (85 F)		130.0 (153L)			
		1500			130.0 (153L)	252.3 (153L)	420.5 (153L)	555.9 (153L)
1.2 (H)	Low 1000	10	-227.5 (488F)	-122.2 (86 F)	106.5 (290L)	198.2 (153L)		
		200	-227.5 (488F)		100.7 (290L)			
		750	-227.5 (488F)	-122.2 (86 F)	100.0 (290L)	190.1 (153L)		513.8 (472F)
	High 10000	200	-227.7 (488F)	-120.7 (346N)	104.0 (290L)	234.6 (153L)		
		750	-227.7 (488F)		103.4 (290L)			
		1500			103.2 (290L)	233.9 (153L)	416.2 (153L)	542.8 (153L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE F.2 MAXIMUM STRESS fy
DUE TO TJC 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-119.8 (8 F)	140.6 (173L)	184.4 (174L)			
		200	-119.8 (8 F)		180.8 (174L)			
		750	-119.8 (8 F)	135.5 (173L)	180.6 (174L)			425.6 (270L)
	High 10000	200	-123.1 (436F)	135.9 (173L)	181.2 (174L)			444.7 (270L)
		750	-122.8 (436F)		181.1 (174L)			
		1500			181.0 (174L)		374.3 (270L)	444.3 (270L)
0.3 (M)	Low 1000	10	-169.3 (475F)	87.6 (173L)	136.2 (173L)	232.3 (270L)		
		200	-169.2 (475F)		131.1 (173L)			
		750	-169.2 (475F)	-84.7 (293C)	129.9 (173L)	226.1 (270L)		416.6 (270L)
	High 10000	200	-169.3 (293C)	-86.4 (293C)	132.5 (173L)	234.9 (270L)		
		750	-169.2 (293C)		131.4 (173L)			
		1500			131.0 (173L)	234.5 (270L)	377.8 (270L)	455.0 (270L)
1.2 (H)	Low 1000	10	-225.7 (160N)	-135.3 (163C)	104.6 (173L)	216.7 (270L)		
		200	-224.6 (293C)		101.1 (173L)			
		750	-224.6 (293C)	-133.9 (163C)	99.8 (173L)	216.9 (153L)		440.0 (270L)
	High 10000	200	-236.2 (176C)	-140.4 (176C)	101.8 (173L)	225.6 (153L)		
		750	-235.9 (176C)		100.6 (173L)			
		1500			100.2 (173L)	226.0 (153L)	391.1 (270L)	497.2 (270L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE F.3 MAXIMUM STRESS fxy
DUE TO TJC 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{O}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-50.2 (507F)	42.2 (327N)	54.7 (327N)			
		200	-50.2 (507F)		54.4 (327N)			
		750	-50.2 (507F)	-41.8 (312N)	54.4 (327N)			150.3 (130N)
	High 10000	200	-49.9 (507F)	-41.5 (312N)	45.2 (327N)			138.6 (507F)
		750	-49.9 (507F)		45.2 (327N)			
		1500			45.2 (327N)		94.1 (507F)	138.6 (507F)
0.3 (M)	Low 1000	10	-37.6 (507F)	-28.5 (146N)	37.0 (315N)	50.3 (130N)		
		200	-37.6 (507F)		36.2 (315N)			
		750	-37.6 (507F)	-30.9 (146N)	36.1 (315N)	52.1 (130N)		148.2 (507F)
	High 10000	200	-37.6 (507F)	-31.0 (146N)	32.6 (315N)	37.4 (315N)		
		750	-37.6 (507F)		32.5 (315N)			
		1500			32.5 (315N)	37.4 (315F)	92.1 (507F)	147.2 (507F)
1.2 (H)	Low 1000	10	-36.2 (7 F)	-22.9 (159N)	23.5 (315N)	37.8 (191C)		
		200	-36.2 (7 F)		23.4 (315N)			
		750	-36.2 (7 F)	-25.5 (160L)	23.4 (315N)	32.7 (265N)		164.2 (507F)
	High 10000	200	-36.2 (7 F)	-23.1 (160L)	23.0 (315N)	31.1 (5 F)		
		750	-36.2 (7 F)		-23.0 (160L)			
		1500			-23.1 (160L)	31.1 (5 F)	99.4 (505F)	163.2 (507F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE F.4 MAXIMUM PRINCIPAL STRESS f1
DUE TO TJC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-118.7 (475F)	144.3 (173L)	186.8 (278L)			
		200	-118.6 (475F)		186.4 (278L)			
		750	-118.5 (475F)	142.8 (173L)	186.1 (278L)			469.3 (166L)
	High 10000	200	-121.2 (462F)	142.5 (173L)	189.2 (278L)			566.6 (153L)
		750	-121.1 (462F)		188.7 (278L)			
		1500			188.6 (278L)		450.8 (153L)	565.8 (153L)
0.3 (M)	Low 1000	10	-169.2 (475F)	91.9 (173L)	137.9 (173L)	234.7 (166N)		
		200	-169.2 (475F)		135.5 (173L)			
		750	-169.2 (475F)	93.1 (173L)	135.1 (303L)	235.0 (166N)		465.1 (153L)
	High 10000	200	-169.1 (475F)	89.0 (173L)	142.3 (173L)	254.9 (153L)		
		750	-169.1 (475F)		142.2 (173L)			
		1500			142.1 (173L)	254.2 (153L)	428.3 (153L)	566.8 (153L)
1.2 (H)	Low 1000	10	-211.0 (98 N)	-111.5 (345N)	108.3 (186L)	220.8 (166N)		
		200	-210.7 (98 N)		105.8 (173L)			
		750	-210.6 (98 N)	-111.1 (345N)	105.6 (173L)	219.3 (166N)		513.8 (153L)
	High 10000	200	-210.9 (345N)	-111.9 (345N)	108.8 (173L)	235.2 (153L)		
		750	-210.9 (345N)		108.9 (173L)			
		1500			108.9 (173L)	234.3 (153L)	428.6 (153L)	558.4 (153L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE F.5 MAXIMUM PRINCIPAL STRESS f2
DUE TO TJC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-225.2 (72 F)	-134.9 (73 F)	119.8 (153L)			
		200	-224.8 (72 F)		119.5 (153L)			
		750	-224.8 (72 F)	-134.8 (73 F)	119.6 (153L)			-345.5 (222N)
	High 10000	200	-227.6 (72 F)	-134.5 (73 F)	157.2 (153L)			444.0 (270L)
		750	-227.6 (72 F)		157.5 (153L)			
		1500			157.6 (153L)		372.1 (270L)	443.2 (270L)
0.3 (M)	Low 1000	10	-227.9 (72 F)	-129.7 (85 F)	107.5 (153L)	176.0 (153L)		
		200	-227.6 (72 F)		108.6 (153L)			
		750	-227.6 (72 F)	-129.4 (85 F)	108.8 (153L)	175.8 (153L)		314.2 (333N)
	High 10000	200	-227.4 (85 F)	-131.0 (85 F)	123.9 (291L)	234.5 (270L)		
		750	-227.4 (85 F)		124.4 (291L)			
		1500			124.5 (291L)	233.3 (270L)	375.9 (270L)	453.3 (270L)
1.2 (H)	Low 1000	10	-227.5 (488F)	-135.3 (163C)	95.7 (290L)	192.4 (153L)		
		200	-227.5 (488F)		98.0 (290L)			
		750	-227.5 (488F)	-134.2 (163C)	98.5 (290L)	188.0 (153L)		359.2 (345N)
	High 10000	200	-236.8 (176C)	-140.6 (176C)	98.9 (290L)	224.9 (153L)		
		750	-236.4 (176C)		99.5 (290L)			
		1500			99.7 (290L)	225.6 (153L)	384.1 (270L)	493.4 (270L)

Temperature differentials dT(^{o}F) apply to the columns -10 through +25.

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE F.6 MAXIMUM PRINCIPAL STRESS Tmax DUE TO TJC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	100.5 (79 F)	71.2 (92 N)	72.7 (212N)			
		200	100.4 (78 F)		66.9 (213N)			
		750	100.5 (78 F)	69.5 (353F)	67.3 (213N)			209.0 (222N)
	High 10000	200	-102.0 (92 N)	69.0 (92 N)	51.5 (147N)			223.6 (222N)
		750	101.8 (92 N)		51.5 (328N)			
		1500			51.5 (328N)		168.4 (222N)	223.5 (222N)
0.3 (M)	Low 1000	10	94.5 (78 F)	57.5 (92 N)	53.3 (212N)	80.5 (213N)		
		200	94.8 (78 F)		46.4 (212N)			
		750	94.8 (78 F)	56.0 (92 N)	44.8 (147N)	75.6 (231N)		207.6 (481F)
	High 10000	200	93.5 (78 F)	57.3 (92 N)	38.6 (147N)	76.6 (130N)		
		750	93.5 (78 F)		38.5 (147N)			
		1500			38.4 (147N)	76.5 (130N)	152.5 (222N)	212 (222N)
1.2 (H)	Low 1000	10	92.4 (215C)	56.8 (202C)	40.5 (212N)	75.6 (231N)		
		200	91.8 (91 F)		33.6 (212N)			
		750	91.9 (228C)	56.2 (228C)	31.0 (199N)	71.5 (482F)		222.6 (469F)
	High 10000	200	89.2 (91 F)	49.1 (91 F)	31.2 (212N)	71.8 (482F)		
		750	89.2 (91 F)		29.2 (147N)			
		1500			29.1 (147N)	71.8 (482F)	156.7 (482F)	213.2 (53 F)

(Column headings -10 to +25: Temperature differentials dT(^{o}F))

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE G.1 MAXIMUM STRESS fx
DUE TO TME 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-203.7 (144N)	159.6 (209L)	244.3 (209L)			
		100	-206.6 (144N)		243.8 (209L)			
		750	-206.9 (144N)	158.9 (209L)	243.7 (209L)			672.6 (209L)
	High 10000	100	-221.1 (131N)	162.3 (209L)	244.5 (209L)			660.1 (209L)
		750	-221.4 (131N)		244.4 (209L)			
		1500			244.4 (209L)		541.6 (209L)	659.9 (209L)
0.3 (M)	Low 1000	10	-215.8 (144N)	-138.8 (144N)	220.6 (209L)	359.8 (209L)		
		200	-217.5 (144N)		220.5 (209L)			
		750	-217.7 (144N)	-140.6 (365N)	220.5 (209L)	359.7 (209L)		662.5 (209L)
	High 10000	200	-222.1 (365N)	-144.7 (365N)	221.2 (209L)	355.1 (209L)		
		750	-222.4 (365N)		221.2 (209L)			
		1500			221.2 (209L)	355.1 (209L)	521.8 (209L)	653.6 (293L)
1.2 (H)	Low 1000	10	-223.1 (358N)	-138.1 (365N)	188.3 (209L)	324.4 (209L)		
		200	-224.1 (358N)		188.4 (209L)			
		750	-224.2 (358N)	-139.1 (365N)	188.4 (209L)	324.5 (209L)		670.4 (293L)
	High 10000	200	-225.9 (358N)	-139.4 (365N)	188.5 (209L)	320.8 (209L)		
		750	-226.0 (358N)		188.5 (209L)			
		1500			188.5 (209L)	320.8 (209L)	499.1 (294L)	654.8 (293L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE G.2 MAXIMUM STRESS fy
DUE TO TME 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-118.5 (8 F)	117 (190L)	173.9 (190L)			
		200	-118.4 (8 F)		173.0 (190L)			
		750	-118.4 (8 F)	115.9 (190L)	172.9 (190L)			466.3 (293L)
	High 10000	200	-120.3 (449F)	113.0 (190L)	172.3 (190L)			495.7 (293L)
		750	-120.2 (449F)		172.2 (190L)			
		1500			172.2 (190L)		399.2 (190L)	495.4 (293L)
0.3 (M)	Low 1000	10	-175.2 (292C)	-102.9 (188N)	124.6 (190L)	268.7 (294L)		
		200	-175.5 (188N)		124.4 (190L)			
		750	-175.5 (188N)	-103.2 (188N)	124.4 (190L)	268.3 (294L)		498.6 (190L)
	High 10000	200	-176.9 (188N)	-103.8 (188N)	124.3 (190L)	269.1 (294L)		
		750	-177.7 (188N)		124.3 (190L)			
		1500			124.3 (190L)	269.0 (294L)	433.7 (190L)	506.6 (190L)
1.2 (H)	Low 1000	10	-243.9 (292C)	-153.9 (292C)	95.5 (294L)	276.2 (294L)		
		200	-243.7 (292C)		95.5 (294L)			
		750	-243.7 (292C)	-153.9 (292C)	95.5 (294L)	275.7 (294L)		554.0 (190L)
	High 10000	200	-243.0 (292C)	-153.7 (292C)	95.6 (294L)	274.3 (294L)		
		750	-243.0 (292C)		95.6 (294L)			
		1500			95.6 (294L)	274.3 (294L)	453.0 (190L)	577.3 (190L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE G.3 MAXIMUM STRESS fxy
DUE TO TME 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-56.0 (377N)	-44.7 (377N)	-42.3 (321N)			
		200	-58.6 (390F)		-42.3 (321N)			
		750	-59.1 (390F)	-45.7 (377N)	-42.3 (321N)			135.7 (507F)
	High 10000	200	-48.8 (507F)	-42.6 (321N)	-42.0 (321N)			132.7 (507F)
		750	-48.8 (507F)		-42.0 (321N)			
		1500			-42.0 (321N)		91.8 (507F)	132.7 (507F)
0.3 (M)	Low 1000	10	44.0 (157N)	45.0 (184N)	46.3 (184N)	46.3 (184N)		
		200	44.1 (184N)		46.4 (184N)			
		750	44.1 (184N)	45.0 (184N)	46.4 (184N)	46.3 (184N)		136.9 (507F)
	High 10000	200	45.7 (184N)	46.1 (184N)	46.6 (184N)	45.2 (184N)		
		750	45.7 (184N)		46.6 (184N)			
		1500			46.6 (184N)	45.2 (184N)	88.7 (507F)	136.2 (507F)
1.2 (H)	Low 1000	10	49.8 (184N)	49.5 (184N)	47.7 (184N)	42.8 (184N)		
		200	49.6 (184N)		47.8 (184N)			
		750	49.6 (184N)	49.5 (184N)	47.8 (184N)	42.0 (184N)		155.9 (507F)
	High 10000	200	50.1 (184N)	49.3 (184N)	47.8 (184N)	44.0 (184N)		
		750	50.1 (184N)		47.8 (184N)			
		1500			47.8 (184N)	44.0 (184N)	105.5 (507F)	155.2 (507F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE G.4 MAXIMUM PRINCIPAL STRESS f1
DUE TO TME 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-117.4 (449F)	160.0 (209L)	244.7 (209L)			
		200	-116.7 (462F)		244.3 (209L)			
		750	-116.7 (462F)	159.3 (209L)	244.2 (209L)			673.3 (209L)
	High 10000	200	-120.3 (449F)	163.1 (209L)	245.0 (209L)			660.2 (209L)
		750	-120.2 (449F)		244.9 (209L)			
		1500			244.9 (209L)		541.9 (209L)	660.1 (209L)
0.3 (M)	Low 1000	10	-168.1 (462F)	136.7 (209L)	221.9 (209L)	360.5 (209L)		
		200	-168.0 (462F)		221.8 (209L)			
		750	-168.0 (462F)	136.6 (209L)	221.8 (209L)	360.4 (209L)		662.8 (209L)
	High 10000	200	-168.3 (462F)	140.8 (209L)	222.5 (209L)	355.7 (209L)		
		750	-168.2 (462F)		222.5 (209L)			
		1500			222.5 (209L)	355.7 (209L)	522.1 (209L)	654.0 (293L)
1.2 (H)	Low 1000	10	-204.4 (149N)	-115.1 (149N)	190.0 (209L)	324.9 (209L)		
		200	-205.0 (149N)		190.1 (209L)			
		750	-205.0 (149N)	-115.5 (149N)	190.1 (209L)	324.9 (209L)		670.7 (293L)
	High 10000	200	-204.7 (149N)	-115.3 (149N)	190.2 (209L)	321.4 (209L)		
		750	-204.7 (149N)		190.2 (209L)			
		1500			190.2 (209L)	321.4 (209L)	501.4 (294L)	654.9 (293L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE G.5 MAXIMUM PRINCIPAL STRESS f2
DUE TO TME 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-204.0 (144N)	-131.2 (131N)	162.4 (190L)			
		200	-206.9 (144N)		161.7 (190L)			
		750	-207.3 (144N)	-135.3 (131N)	161.6 (190L)			465.8 (293L)
	High 10000	200	-221.8 (131N)	-145.5 (131N)	160.9 (190L)			495.1 (293L)
		750	-222.1 (131N)		160.8 (190L)			
		1500			160.8 (190L)		398.0 (190L)	494.7 (293L)
0.3 (M)	Low 1000	10	-216.3 (144N)	-140.5 (365N)	120.7 (190L)	266.5 (190L)		
		200	-218.0 (144N)		120.5 (190L)			
		750	-218.0 (144N)	-142.6 (365N)	120.4 (190L)	266.0 (190L)		497.9 (190L)
	High 10000	200	-223.7 (365N)	-147.1 (365N)	120.5 (190L)	267.0 (190L)		
		750	-223.9 (365N)		120.4 (190L)			
		1500			120.4 (190L)	267.0 (190L)	432.8 (190L)	505.9 (190L)
1.2 (H)	Low 1000	10	-244.2 (292C)	-154.2 (292C)	95.0 (190L)	271.4 (190L)		
		200	-244.0 (292C)		95.0 (190L)			
		750	-244.0 (292C)	-154.2 (292C)	95.0 (190L)	271.2 (190L)		554.0 (190L)
	High 10000	200	-243.4 (292C)	-154.0 (292C)	95.1 (190L)	270.6 (190L)		
		750	-243.4 (292C)		95.1 (190L)			
		1500			95.1 (190L)	270.6 (190L)	452.7 (190L)	577.1 (190L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE G.6 MAXIMUM PRINCIPAL STRESS Tmax
DUE TO TME 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	105.5 (131N)	71.9 (131N)	99.9 (313L)			
		200	107.8 (131N)		99.6 (313L)			
		750	108.2 (131N)	74.1 (131N)	99.5 (313L)			305.4 (313L)
	High 10000	200	115.2 (131N)	78.3 (131N)	99.9 (313L)			300.3 (313L)
		750	115.3 (131N)		99.9 (313L)			
		1500			99.8 (313L)		243.0 (313L)	300.2 (313L)
0.3 (M)	Low 1000	10	110.5 (157N)	75.1 (157N)	86.8 (313L)	153.4 (313L)		
		200	111.0 (157N)		86.7 (313L)			
		750	111.0 (157N)	75.5 (157N)	86.7 (313L)	153.4 (313L)		299.9 (313L)
	High 10000	200	113.4 (144N)	76.6 (157N)	87.0 (313L)	151.2 (313L)		
		750	113.4 (144N)		87.0 (313L)			
		1500			87.0 (313L)	151.2 (313L)	231.5 (313L)	291.5 (313L)
1.2 (H)	Low 1000	10	112.9 (157N)	78.5 (157N)	68.3 (313L)	133.2 (313L)		
		200	113.3 (157N)		68.3 (313L)			
		750	113.3 (157N)	78.6 (157N)	68.3 (313L)	133.0 (313L)		289.2 (313L)
	High 10000	200	113.1 (157N)	77.6 (157N)	68.4 (313L)	131.6 (313L)		
		750	113.1 (157N)		68.4 (313L)			
		1500			68.4 (313L)	131.6 (313L)	215.1 (313L)	273.7 (313L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE H.1 MAXIMUM STRESS fx
DUE TO TMC 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-189.6 (462F)	-95.4 (144N)	183.9 (309L)			
		200	-189.6 (462F)		183.7 (309L)			
		750	-189.6 (462F)	-98.0 (144N)	183.7 (309L)			651.1 (226L)
	High 10000	200	-196.2 (135N)	-115.1 (144N)	183.1 (309L)			650.9 (226L)
		750	-196.2 (135N)		183.1 (309L)			
		1500			183.1 (309L)		515.2 (226L)	651.1 (226L)
0.3 (M)	Low 1000	10	-206.8 (46 F)	-112.2 (148N)	142.2 (309L)	304.8 (226L)		
		200	-206.8 (46 F)		142.1 (309L)			
		750	-206.8 (46 F)	-113.0 (148N)	142.1 (309L)	305.0 (226L)		653.3 (226L)
	High 10000	200	-212.1 (137N)	-121.1 (136N)	142.4 (309L)	301.2 (309L)		
		750	-212.2 (137N)		142.4 (309L)			
		1500			142.4 (309L)	301.2 (309L)	497.0 (309L)	643.4 (309L)
1.2 (H)	Low 1000	10	-217.8 (150N)	-122.1 (149N)	106.1 (225L)	277.3 (205L)		
		200	-219.1 (150N)		106.1 (225L)			
		750	-219.2 (150N)	-122.5 (149N)	106.1 (225L)	279.2 (309L)		662.0 (226L)
	High 10000	200	-220.9 (150N)	-122.1 (149N)	106.2 (225L)	271.4 (309L)		
		750	-221.0 (150N)		106.2 (225L)			
		1500			106.2 (225L)	271.4 (309L)	485.5 (205L)	636.7 (309L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE H.2 MAXIMUM STRESS fy
DUE TO TMC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-118.5 (8 F)	138.6 (329L)	182.2 (226L)			
		200	-118.4 (8 F)		181.5 (226L)			
		750	-118.4 (8 F)	137.7 (329L)	181.4 (226L)			(309L)
	High 10000	200	-120.0 (59 F)	135.5 (329L)	181.0 (226L)			451.0 (309L)
		750	-119.8 (59 F)		180.9 (226L)			
		1500			180.9 (226L)		373.2 (309L)	450.9 (309L)
0.3 (M)	Low 1000	10	-170.3 (215C)	-85.5 (215C)	131.5 (329L)	246.9 (309L)		
		200	-170.7 (215C)		131.3 (329L)			
		750	-170.7 (215C)	-85.8 (332C)	131.3 (329L)	246.6 (309L)		464.6 (309L)
	High 10000	200	-172.2 (332C)	-86.7 (332C)	131.2 (329L)	247.3 (309L)		
		750	-172.2 (332C)		131.2 (225L)			
		1500			131.2 (225L)	247.3 (309L)	390.5 (309L)	465.6 (309L)
1.2 (H)	Low 1000	10	-246.4 (228C)	-144.7 (228C)	100.0 (225L)	244.0 (309L)		
		200	-246.3 (228C)		100.0 (225L)			
		750	-246.3 (228C)	-144.7 (228C)	100.0 (225L)	244.0 (309L)		515.9 (309L)
	High 10000	200	-245.4 (228C)	-144.3 (228C)	100.1 (225L)	239.5 (309L)		
		750	-245.4 (228C)		100.1 (225L)			
		1500			100.1 (225L)	239.5 (309L)	408.4 (205L)	518.8 (309L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE H.3 MAXIMUM STRESS fxy
DUE TO TMC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	-10	-5	0	+8	+18	+25
			Temperature differentials dT(^{o}F)					
0.1 (S)	Low 1000	10	-62.5 (390F)	-50.4 (377N)	-42.7 (351N)			
		200	-66.4 (390F)		-43.2 (351N)			
		750	-67.1 (390F)	-52.2 (377N)	-43.3 (351N)			135.6 (507F)
	High 10000	200	-48.7 (507F)	-44.0 (351N)	-41.8 (351N)			132.6 (507F)
		750	-48.7 (507F)		41.8 (351N)			
		1500			-41.9 (351N)		91.8 (507F)	132.6 (507F)
0.3 (M)	Low 1000	10	-38.5 (505F)	-32.9 (351N)	-31.2 (198N)	-32.6 (198N)		
		200	-40.8 (414F)		31.2 (354N)			
		750	-41.3 (414F)	-32.9 (351N)	31.3 (354N)	32.8 (354N)		136.8 (507F)
	High 10000	200	-38.5 (505F)	31.4 (354N)	31.1 (354N)	33.3 (354N)		
		750	-38.5 (505F)		31.2 (354N)			
		1500			31.2 (354N)	33.5 (354N)	88.7 (507F)	136.1 (507F)
1.2 (H)	Low 1000	10	-34.8 (504F)	-22.3 (212L)	-23.4 (212L)	31.1 (505F)		
		200	-34.8 (504F)		-23.4 (212L)			
		750	-34.8 (504F)	-22.4 (212L)	-23.4 (212L)	-31.6 (323N)		155.9 (507F)
	High 10000	200	-34.8 (504F)	-22.5 (212L)	-23.4 (212L)	31.5 (4 F)		
		750	-34.8 (504F)		-23.4 (212L)			
		1500			-23.4 (212L)	31.5 (4 F)	105.5 (507F)	155.1 (507F)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9", Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE H.4 MAXIMUM PRINCIPAL STRESS f1
DUE TO TMC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Joint Stiffness Kl	Temperature differentials dT(^{o}F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-117.4 (59 F)	144.8 (225L)	194.8 (309L)			
		200	-116.7 (46 F)		194.5 (309L)			
		750	-116.6 (46 F)	144.1 (225L)	194.4 (309L)			651.7 (226L)
	High 10000	200	-120.0 (59 F)	142.0 (225L)	193.6 (309L)			651.8 (226L)
		750	-119.8 (59 F)		193.6 (309L)			
		1500			193.6 (309L)		516.2 (226L)	652.0 (226L)
0.3 (M)	Low 1000	10	-168.1 (46 F)	85.8 (225L)	148.7 (225L)	306.2 (226L)		
		200	-168.0 (46 F)		148.7 (225L)			
		750	-168.0 (46 F)	85.6 (225L)	148.7 (225L)	306.4 (226L)		654.3 (226L)
	High 10000	200	-168.2 (46 F)	85.7 (225L)	149.0 (225L)	302.6 (309L)		
		750	-168.2 (46 F)		149.0 (225L)			
		1500			149.0 (225L)	302.6 (309L)	497.6 (309L)	644.5 (226L)
1.2 (H)	Low 1000	10	-213.4 (150N)	-114.0 (150N)	111.6 (225L)	279.8 (309L)		
		200	-214.0 (150N)		111.7 (225L)			
		750	-214.0 (150N)	-114.2 (150N)	111.7 (225L)	282.3 (309L)		664.0 (226L)
	High 10000	200	-214.3 (150N)	-114.7 (371N)	111.8 (225L)	273.2 (309L)		
		750	-214.3 (150N)		111.8 (225L)			
		1500			111.8 (225L)	273.2 (309L)	485.5 (205L)	638.1 (309L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE H.5 MAXIMUM PRINCIPAL STRESS f2
DUE TO TMC 40 KIPS AXLE LOADING

Sub	Joint Stiffness		Temperature differentials dT(^{o}F)					
Ks	Kr	Kl	-10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	-189.6 (462F)	-114.7 (144N)	176.2 (330L)			
		200	-189.6 (462F)		175.2 (205L)			
		750	-189.6 (462F)	-117.4 (144N)	175.1 (205L)			435.3 (309L)
	High 10000	200	-200.0 (134F)	-120.8 (132N)	174.9 (205L)			450.4 (309L)
		750	-200.1 (134F)		174.8 (205L)			
		1500			174.8 (205L)		372.4 (309L)	450.2 (309L)
0.3 (M)	Low 1000	10	-206.8 (46 F)	-114.4 (147N)	128.7 (329L)	245.7 (309L)		
		200	-206.8 (46 F)		128.3 (329L)			
		750	-206.8 (46 F)	-115.2 (147N)	128.2 (329L)	245.3 (309L)		464.1 (309L)
	High 10000	200	-212.1 (137N)	-121.7 (136N)	128.3 (329L)	246.0 (309L)		
		750	-212.2 (137N)		128.2 (329L)			
		1500			128.2 (329L)	246.0 (309L)	389.9 (309L)	464.7 (309L)
1.2 (H)	Low 1000	10	-246.4 (228C)	-144.7 (228C)	99.5 (329L)	242.7 (205L)		
		200	-246.3 (228C)		99.4 (329L)			
		750	-246.3 (228C)	-144.7 (228C)	99.4 (329L)	242.4 (205L)		514.7 (309L)
	High 10000	200	-245.4 (228C)	-144.3 (228C)	99.5 (329L)	238.5 (205L)		
		750	-245.4 (228C)		99.5 (329L)			
		1500			99.5 (329L)	238.5 (205L)	408.3 (205L)	517.4 (309L)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

TABLE H.6 MAXIMUM PRINCIPAL STRESS Tmax
DUE TO TMC 40 KIPS AXLE LOADING

Sub Ks	Joint Stiffness Kr	Kl	dT(°F) -10	-5	0	+8	+18	+25
0.1 (S)	Low 1000	10	104.2 (157N)	70.0 (157N)	45.7 (159N)			
		200	105.1 (157N)		46.2 (363N)			
		750	105.3 (157N)	70.8 (157N)	46.3 (363N)			239.4 (261N)
	High 10000	200	102.4 (157N)	68.2 (145N)	46.2 (367N)			231.5 (260N)
		750	102.5 (157N)		46.4 (367N)			
		1500			46.4 (367N)		174.4 (273N)	231.5 (260N)
0.3 (M)	Low 1000	10	96.3 (157N)	59.6 (157N)	35.3 (199N)	87.7 (273N)		
		200	96.6 (157N)		35.3 (199N)			
		750	96.6 (157N)	59.8 (157N)	35.3 (199N)	87.7 (273N)		234.8 (261N)
	High 10000	200	95.2 (157N)	58.7 (157N)	35.1 (199N)	85.4 (273N)		
		750	95.2 (157N)		35.1 (199N)			
		1500			35.1 (199N)	85.4 (273N)	166.8 (273N)	225.9 (261N)
1.2 (H)	Low 1000	10	89.9 (157N)	50.8 (157N)	28.7 (306C)	77.1 (169N)		
		200	90.5 (157N)		28.7 (306C)			
		750	90.5 (157N)	50.9 (157N)	28.7 (306C)	76.8 (169N)		228.1 (248N)
	High 10000	200	89.0 (157N)	50.0 (157N)	28.8 (306C)	74.8 (169N)		
		750	89.1 (157N)		28.8 (306C)			
		1500			28.8 (306C)	74.9 (169N)	156.6 (169N)	215.6 (273N)

Input Data : Ke = 10 ,Skew = 9.46 thk = 9" , Ec = 5290 ksi
Notations : S = Soft, M = Medium, H = Hard (Econocrete)
Ks = Spring stiffness of subgrade.
Kr = Rotational spring stiffness of Joint.
Kl = Linear spring stiffness of Joint.
dT = Temperature differential (Top - Bottom)

APPENDIX B

STIFFNESS MATRIX OF PARALLELOGRAM PLATE BENDING ELEMENT

```
C-------------------------------------------------------------
C|  SUBROUTINE KMAT WILL INITIALIZE THE ELEMENTAL STIFFNESS
C   MATRIX USING THE [SK1], [SK2], [SK3], AND [SK4] MATRICES
C   THAT ARE INITIALIZED BY INTERNAL DATA STATEMENTS.
C   MATRICES [SK1] AND [SK2] HAVE BEEN ADJUSTED FOR THE
C   WIDTH AND LENGTH OF THE ELEMENT.
C-------------------------------------------------------------
C
        SUBROUTINE KMAT (SK,ELEN,EWID)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION SK(12,12)
        INTEGER BW
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
     COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
     +                  NOE,NDISP,NNOD,BW,NBK,IRUN
C
        DO 10 I=1,11
      DO 10 J=I+1,12
   10    SK(I,J) = 0.0
C--------------------------------------------------------
C   DEFINE THE CONSTANTS USED FOR STIFFNESS MATRIX
C--------------------------------------------------------
C
      A = ELEN * 0.5
      B = EWID * 0.5
C
      A1 = 1./ A
      A2 = 1./(A * A)
      A3 = 1./(A * A * A)
      A4 = 1./(A * A * A * A)
        B1 = 1./ B
      B2 = 1./(B * B)
      B3 = 1./(B * B * B)
      B4 = 1./(B * B * B * B)
C
      COT = 1./ TAN(ANG)
      CSC = 1./ SIN(ANG)
      IF(ANGLE.EQ.90.) COT=0.0
C
      COT2 = COT**2
        COT3 = COT**3
      COT4 = COT**4
      CSC2 = CSC**2
      CSC3 = CSC**3
      CSC4 = CSC**4
C
C---( for SK2 )
C
      C1C1 = A4 * COT4
      C2C2 = B4
      C3C3 = A2 * B2 * COT2
```

```
C
      C6C6 = A2 * COT2 * CSC2
      C7C7 = B2 * CSC2
      C6C7 = A1 * B1 * COT * CSC2
C
      C1C7 = A2 * B1 * COT2 * CSC
      C2C6 = A1 * B2 * COT *CSC
      C2C7 = B3 * CSC
C
      C6C9 = A2 * COT3 * CSC
      C7C9 = A1 * B1 * COT2 * CSC
      C7C10= B2 * COT * CSC
      C9C9 = A2 * COT4
      C10C10 = B2 * COT2
      C9C10 = A1 * B1 * COT3
C
      C1C9 = A3 * COT4
      C1C10 = A2 * B1 * COT3
      C2C9 = A1 * B2 * COT2
      C2C10 = B3 * COT
C
C---( for SK3 )
C
      C1 = A4 * COT2
      C2 = A2 * B2
      C7 = A2 * B1 * CSC
C
      C1D = A3 * COT2
      C2D = A1 * B2
      C3D = A2 * B1 * COT
      DC6 = A2 * COT * CSC
      DC7 = A1 * B1 * CSC
      Z4 = A2 * COT2
      Z5 = A1 * B1 * COT
C
C---( for SK4 )
C
      H44 = A4 * COT2
      H55 = A2 * B2
      H45 = A3 * B1 * COT
      H48 = A3 * COT * CSC
      H58 = A2 * B1 * CSC
C
      P88 = A2 * CSC2
      P81 = A2 * COT * CSC
      P82 = A1 * B1 * CSC
      Q11 = A2 * COT2
      Q22 = B2
      Q12 = A1 * B1 * COT
C
C------------------------------------------------------------
C
```

```
C       ELEMENT  STIFFNESS MATRIX
C
CSK = ab.sin(a) [ SK1.Dx + SK2.Dy + SK3.D1 + SK4.Dxy ]
C
C        for isotrofic material,    Dx = Dy = 1
C                                 D1 = PR(poisson's ratio)
C                                      Dxy = (1. - PR ) /2.
C
C               multiplication factor, S =  ab.sin(a) * D
C
C                                      where a= ELEN/2.
C                                       b= EWID/(sin(a)*2.)
C                                     D= E*T**3/12(1-PR**2)
C
C                                          ELEN*EWID*E*T**3
C                                      S = ----------------
C                                          48.* (1.- PR**2)
C
C-----------------------------------------------------------
C
C
      DXY = (1.-PR)/2.
C
SK(1,1) = A4 + (C1C1+C2C2+1.9*C3C3) + PR*(2*C1+0.5*C2)
     *                + DXY*(4*H44+(7./5.)*H55)
SK(2,1) = (-0.7*C1C7-C2C7) + PR*(-0.5*C7)+DXY*(-0.2*H58)
SK(2,2) = 0. + ((8./15.)*C6C6+(4./3.)*C7C7-C6C7) + 0.
     *                + DXY*(8./15.)*P88
SK(3,1) = A3 + (C1C9+0.7*C2C9) + PR*(2.*C1D+0.5*C2D)
     *                + DXY*(4.*C1D+0.2*C2D)
SK(3,2) = ((1./6.)*C6C9-C7C9+(1./6.)*C7C10)+ PR*((1./6.)
     *             *DC6-DC7) + DXY*(1./3.)*P81
SK(3,3) = (4./3.)*A2 + ((4./3.)*C9C9+(8./15.)*C10C10-(1.)*
   *     C9C10) + PR*((8./3.)*Z4-Z5) + DXY*((16./3.)*Q11
     *                + (8./15.)*Q22-2.*Q12)
SK(4,1) = 0.5*A4 + (0.5*C1C1-C2C2-1.9*C3C3)
     * + PR*(C1-0.5*C2)      + DXY*(2.*H44-(7./5.)*H55)
SK(4,2) = 0. + (0.2*C1C7+C2C7) + 0. + DXY*0.2*H58
SK(4,3) = 0.5*A3 + (0.5*C1C9-C1C10-0.7*C2C9)
   * + PR*(C1D-C3D-0.5*C2D) + DXY*(2.*C1D-2.*C3D-0.2*C2D)
SK(4,4) = A4 + (C1C1+C2C2+1.9*C3C3) + PR*(2.*C1+0.5*C2)
     *                + DXY*(4*H44+(7./5.)*H55)
SK(5,1) = 0. + (-0.2*C1C7-C2C7) + 0. + DXY*(-0.2*H58)
SK(5,2) = 0. + ((-2./15.)*C6C6+(2./3.)*C7C7) + 0.
     *                + DXY*(-2./15.)*P88
SK(5,3) = (-1./6.)*C6C9-(1./6.)*C7C10 + PR*(-1./6.)*DC6
     *                + DXY*(-1./3.)*P81
SK(5,4) = (0.7*C1C7+C2C7) + PR*0.5*C7 + DXY*0.2*H58
SK(5,5) = 0. + ((8./15)*C6C6+(4./3.)*C7C7+C6C7) + 0.
     *                + DXY*(8./15.)*P88
SK(6,1) = 0.5*A3 + (0.5*C1C9+C1C10-0.7*C2C9) + PR*(C1D-
     * 0.5*C2D+ C3D) + DXY*(2.*C1D+2.*C3D-0.2*C2D)
```

```
SK(6,2) = (-1./6.)*C6C9-(1./6.)*C7C10 + PR*(-1./6.)*DC6
     *                + DXY*(-1./3.)*P81
SK(6,3) = (2./3.)*A2 + ((2./3.)*C9C9-(8./15)*C10C10) + PR*
     *      (4./3.)*Z4 + DXY*((8./3.)*Q11-(8./15.)*Q22)
SK(6,4) = A3 + (C1C9+0.7*C2C9) + PR*(2.*C1D+0.5*C2D)
     *                + DXY*(4.*C1D+0.2*C2D)
SK(6,5) = 0. + ((1./6.)*C6C9+C7C9+(1./6.)*C7C10)
     *          + PR*((1./6.)*DC6+DC7) + DXY*(1./3.)*P81
SK(6,6) =(4./3.)*A2 + ((4./3.)*C9C9+(8./15.)*C10C10+C9C10)
     *  + PR*((8./3.)*Z4+Z5) + DXY*((16./3.)*Q11+(8./15.)
     *                  *Q22+2.*Q12)
SK(7,1) = -A4 + (-C1C1+0.5*C2C2-1.9*C3C3) + PR*(-2.*C1-
     * 0.5*C2) + DXY*(-4.*H44-(7./5.)*H55)
SK(7,2) = 0. + (0.7*C1C7+C2C6-0.5*C2C7) + PR*(0.5)*C7
     *                + DXY*(0.2*H58)
SK(7,3) = -A3 + (-C1C9-0.2*C2C9) + PR*(-2.*C1D)
     *                + DXY*(-4.*C1D-0.2*C2D)
SK(7,4) = -0.5*A4 + (-0.5*C1C1-0.5*C2C2+1.9*C3C3) +
     * PR*(-C1 + 0.5*C2) + DXY*(-2.*H44+(7./5.)*H55)
SK(7,5) = (0.2*C1C7-C2C6-0.5*C2C7) + 0. + DXY*0.2*H58
SK(7,6) = -0.5*A3 + (-0.5*C1C9-C1C10+0.2*C2C9) + PR*
     * (-C1D-C3D) + DXY*(-2.*C1D-2.*C3D+0.2*C2D)
SK(7,7) = A4 + (C1C1+C2C2+1.9*C3C3) + PR*(2.*C1+0.5*C2)
    *                + DXY*(4.*H44+(7./5.)*H55)
SK(8,1) =(0.7*C1C7-C2C6-0.5*C2C7) + PR*0.5*C7 + DXY*0.2
     *                  *H58
SK(8,2) = 0. + ((-8./15.)*C6C6+(2./3.)*C7C7) + 0.
     *                + DXY*(-8./15.)*P88
SK(8,3) = 0. + (-1./6.)*C6C9-(1./6.)*C7C10 + PR*(-1./6.)
     *                  *DC6 + DXY*(-1./3.)*P81
SK(8,4) =(-0.2*C1C7+C2C6+0.5*C2C7) + 0. + DXY*(-0.2)*H58
SK(8,5) = 0. + ((2./15.)*C6C6+(1./3.)*C7C7+C6C7) + 0.
     *                + DXY*(2./15.)*P88
SK(8,6) = 0. + (1./6.)*C6C9+(1./6.)*C7C10 + PR*(1./6.)
     *                  *DC6 + DXY*(1./3.)*P81
SK(8,7) = 0. + (-0.7*C1C7-C2C7) + PR*(-0.5*C7)
     *                + DXY*(-0.2*H58)
SK(8,8) = 0. + ((8./15.)*C6C6+(4./3.)*C7C7+C6C7) + 0.
     *                + DXY*(8./15.)*P88
SK(9,1) = A3 + (C1C9+0.2*C2C9) + PR*2.*C1D + DXY*
     * (4.*C1D+0.2 *C2D)
SK(9,2) = 0. + (-1./6.)*C6C9+(-1./6.)*C7C10
     *                + PR*(-1./6.)*DC6 + DXY*(-1./3.)*P81
SK(9,3) = (2./3.)*A2 + ((2./3.)*C9C9-(2./15.)*C10C10) + PR*
     *   (4./3.)*Z4 + DXY*((8./3.)*Q11-(2./15.)*Q22)
SK(9,4) = 0.5*A3 + (0.5*C1C9+C1C10-0.2*C2C9) + PR*(C1D+C3D)
     *                + DXY*(2*C1D+2*C3D-0.2*C2D)
        SK(9,5) = 0. + (1./6.)*C6C9+(1./6.)*C7C10
     *                + PR*(1./6.)*DC6 + DXY*(1./3.)*P81
SK(9,6) =(1./3.)*A2 + ((1./3.)*C9C9+(2./15.)*C10C10+C9C10)
     *     + PR*((2./3.)*Z4+Z5) + DXY*((4./3.)*Q11+(2./15.)
     *                  *Q22+2.*Q12)
```

```
SK(9,7) = -A3 + (-C1C9-0.7*C2C9) + PR*(-2.*C1D-0.5*C2D)
     *                 + DXY*(-4.*C1D-0.2*C2D)
SK(9,8) = 0. + ((1./6.)*C6C9+C7C9+(1./6.)*C7C10)
     *            + PR*((1./6.)*DC6+DC7) + DXY*(1./3.)*P81
SK(9,9) = (4./3.)*A2 + ((4./3.)*C9C9+(8./15.)*C10C10+C9C10)
     *    + PR*((8./3.)*Z4+Z5) + DXY*((16./3.)*Q11+(8./15.)
     *                  *Q22+2.*Q12)
SK(10,1) =-0.5*A4 + (-0.5*C1C1-0.5*C2C2+1.9*C3C3) + PR*(-C1
     *               + 0.5*C2) + DXY*(-2.*H44+(7./5.)*H55)
SK(10,2) =(-0.2*C1C7-C2C6+0.5*C2C7) + 0. + DXY*(-0.2*H58)
SK(10,3) = -0.5*A3 + (-0.5*C1C9+C1C10+0.2*C2C9) + PR*(-C1D
     *                 +C3D) + DXY*(-2.*C1D+2.*C3D+0.2*C2D)
SK(10,4) = -A4 + (-C1C1+0.5*C2C2-1.9*C3C3) + PR*(-2.*C1-
     * 0.5*C2) + DXY*(-4.*H44-(7./5.)*H55)
SK(10,5) = 0. + (-0.7*C1C7+C2C6+0.5*C2C7) + PR*(-0.5*C7)
     *                 + DXY*(-0.2*H58)
SK(10,6) = -A3 + (-C1C9-0.2*C2C9) + PR*(-2.*C1D) + DXY*
     *                 (-4.*C1D-0.2*C2D)
SK(10,7) = 0.5*A4 + (0.5*C1C1-C2C2-1.9*C3C3) + PR*(C1-
     * 0.5*C2) + DXY*(2.*H44-(7./5.)*H55)
SK(10,8) = 0. + (0.2*C1C7+C2C7) + 0. + DXY*0.2*H58
SK(10,9) = -0.5*A3 + (-0.5*C1C9-C1C10+0.7*C2C9) + PR*
     * (-C1D-C3D+0.5*C2D) + DXY*(-2.*C1D-2.*C3D+0.2*C2D)
SK(10,10) = A4 + (C1C1+C2C2+1.9*C3C3) + PR*(2.*C1+0.5*C2)
     *                 + DXY*(4.*H44+(7./5.)*H55)
SK(11,1) =(0.2*C1C7+C2C6-0.5*C2C7) + 0. + DXY*(0.2*H58)
SK(11,2) = 0. + ((2./15.)*C6C6+(1./3.)*C7C7-C6C7) + 0.
     *                 + DXY*(2./15.)*P88
SK(11,3) = 0. + (1./6.)*C6C9+(1./6.)*C7C10
     *                 + PR*(1./6.)*DC6 + DXY*(1./3.)*P81
SK(11,4) = 0. + (-0.7*C1C7-C2C6+0.5*C2C7) + PR*(-0.5*C7)
     *                 + DXY*(-0.2*H58)
SK(11,5) =  ((-8./15.)*C6C6+(2./3.)*C7C7) + 0. + DXY*
     *                   (-8./15.)*P88
SK(11,6) = 0. + (-1./6.)*C6C9-(1./6.)*C7C10
     *              + PR*(-1./6.)*DC6 + DXY*(-1./3.)*P81
SK(11,7) = 0. + (-0.2*C1C7-C2C7) + 0. + DXY*(-0.2*H58)
SK(11,8) =  ((-2./15.)*C6C6+(2./3.)*C7C7) + 0. + DXY*
     *                   (-2./15)*P88
SK(11,9) = (-1./6.)*C6C9-(1./6.)*C7C10+ PR*(-1./6.)*DC6
     *                 + DXY*(-1./3.)*P81
SK(11,10) =(0.7*C1C7+C2C7) + PR*(0.5*C7) + DXY*(0.2*H58)
SK(11,11) =0. + ((8./15.)*C6C6+(4./3.)*C7C7-C6C7) + 0.
     *                 + DXY*(8./15.)*P88
SK(12,1) = 0.5*A3 + (0.5*C1C9-C1C10-0.2*C2C9) + PR*(C1D-
     * C3D) + DXY*(2.*C1D-2.*C3D-0.2*C2D)
SK(12,2) = 0. + (1./6.)*C6C9+(1./6.)*C7C10
     *                 + PR*(1./6.)*DC6 + DXY*(1./3.)*P81
SK(12,3) =(1./3.)*A2+((1./3.)*C9C9+(2./15.)*C10C10-C9C10)
     *    + PR*((2./3.)*Z4-Z5) + DXY*((4./3.)*Q11+(2./15.)
     *                 *Q22-2.*Q12)
SK(12,4) =A3+(C1C9+0.2*C2C9) + PR*(2.*C1D) + DXY*(4.*C1D
```

```
     *                   + 0.2*C2D)
SK(12,5) = 0. + (-1./6.)*C6C9-(1./6.)*C7C10
     *              + PR*(-1./6.)*DC6 + DXY*(-1./3.)*P81
SK(12,6) =(2./3.)*A2+((2./3.)*C9C9-(2./15.)*C10C10)+PR*
     *            (4./3.)*Z4 + DXY*((8./3.)*Q11-(2./15.)*Q22)
SK(12,7) = -0.5*A3 + (-0.5*C1C9+C1C10+0.7*C2C9) + PR*(-C1D
     *         + 0.5*C2D+C3D) + DXY*(-2.*C1D+2.*C3D+0.2*C2D)
SK(12,8) = 0. + (-1./6.)*C6C9-(1./6.)*C7C10
     *                   + PR*(-1./6.)*DC6 + DXY*(-1./3.)*P81
SK(12,9) =(2./3.)*A2 + ((2./3.)*C9C9-(8./15)*C10C10) + PR*
     *          (4./3.)*Z4 + DXY*((8./3.)*Q11-(8./15.)*Q22)
SK(12,10)= -A3 + (-C1C9-0.7*C2C9) + PR*(-2.*C1D-0.5*C2D)
     *                    + DXY*(-4.*C1D-0.2*C2D)
SK(12,11)= 0. + ((1./6.)*C6C9-C7C9+(1./6.)*C7C10)
     *                       + PR*((1./6.)*DC6-DC7)
     *                   + DXY*(1./3.)*P81
      SK(12,12)= SK(3,3)
C
C
      DO 100 I=1,12
      DO 100 J=1,I
         IF(I.EQ.J) GO TO 100
         SK(J,I) = SK(I,J)
  100   CONTINUE
C
      D = E*T*T*T/(12.*(1.-PR*PR))
      S = D*ELEN*EWID/4.
C
      CALL SMULT(SK,S,SK,12,12)
        RETURN
        END
```

APPENDIX C

STRESS MATRIX OF PARALLELOGRAM PLATE BENDING ELEMENT

```
C-------------------------------------------------------------
C|   SUBROUTINE STRMAT WILL INITIALIZE THE 12 X 12 STRESS
C    MATRIX [SMAT] WHICH IS COMBINATION OF THE TRANSFORMATION
C    MATRIX [B] AND ELASTICITY MATRIX [D] FOR EACH OF ELEMENT.
C    [SMAT] IS DEPENDENT ON THE ELEMENT'S LENGTH, WIDTH, AND
C    POISSON`S RATIO.
C|
C|   ELEN  = LENGTH OF ELEMENT.
C|   EWID  = WIDTH OF ELEMENT.
C|   A     = ELEN / 2
C|   B     = EWID / (2.*sin(a))
C|   PR    = POISSON'S RATIO
C|   DXY   = (1 - PR)/2
C-------------------------------------------------------------
C
      SUBROUTINE STRMAT(SMAT,ELEN,EWID,E,T,PR,ANGLE,ANG)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION SMAT(12,12)
C
C---------------------------
C     DEFINE THE CONSTANTS
C---------------------------
C
      A = ELEN/2.
      B = EWID/2.
      A1= 1./A
      A2= A1 * A1
      B1= SIN(ANG)/B
      B2= B1 * B1
      D = E*T*T*T/(12.*(1.-PR*PR))
      DXY=(1.-PR)/2.
C
      COT=1./TAN(ANG)
      CSC=1./SIN(ANG)
      IF(ANGLE.EQ.90.) COT=0.0
      COT2=COT*COT
      CSC2=CSC*CSC
C
      C1 = A2*COT2
      C2 = B2*CSC2
      C3 = A1*B1*COT*CSC
      C4 = A2*COT
      C5 = A1*B1*CSC
      C6 = A1*COT*CSC
      C7 = B1*CSC2
      C8 = A1*CSC
      C9 = A1*COT2
      C10= B1*COT*CSC
      C11= A1*COT
      C12= B1*CSC
C
```

```
      CALL ZERO(SMAT,12,12)
C
      SMAT(1,1) = 1.5*A2 + PR*(1.5*C1+1.5*C2-0.5*C3)
      SMAT(1,2) = PR*(C6-2.*C7)
      SMAT(1,3) = 2.*A1 + PR*(2.*C9-C10)
      SMAT(1,4) = PR*(-1.5*C2+0.5*C3)
      SMAT(1,5) = PR*(-C7)
      SMAT(1,6) = PR*(2.*C9+C10)
      SMAT(1,7) = -1.5*A2 + PR*(-1.5*C1+0.5*C3)
      SMAT(1,8) = PR*(-C6)
      SMAT(1,9) = A1 + PR*C9
      SMAT(1,10)= PR*(-0.5*C3)
C
      SMAT(2,1) = PR*(1.5*A2) + 1.5*C1+1.5*C2-0.5*C3
      SMAT(2,2) = C6-2.*C7
      SMAT(2,3) = PR*2.*A1 + 2.*C9-C10
      SMAT(2,4) = -1.5*C2+0.5*C3
      SMAT(2,5) = -C7
      SMAT(2,6) = 2.*C9+C10
      SMAT(2,7) = PR*(-1.5*A2) + (-1.5*C1+0.5*C3)
      SMAT(2,8) = -C6
      SMAT(2,9) = PR*A1 + C9
      SMAT(2,10)= -0.5*C3
C
      SMAT(3,1) = DXY*(3.*C4-0.5*C5)
      SMAT(3,2) = DXY*C8
      SMAT(3,3) = DXY*(4.*C11-C12)
      SMAT(3,4) = DXY*(0.5*C5)
      SMAT(3,6) = DXY*C12
      SMAT(3,7) = DXY*(-3.*C4+0.5*C5)
      SMAT(3,8) = DXY*(-C8)
      SMAT(3,9) = DXY*2.*C11
      SMAT(3,10)= DXY*(-0.5*C5)
C
      SMAT(4,1) = PR*(-1.5*C2-0.5*C3)
      SMAT(4,2) = PR*C7
      SMAT(4,3) = PR*(-C10)
      SMAT(4,4) = 1.5*A2 + PR*(1.5*C1+1.5*C2+0.5*C3)
      SMAT(4,5) = PR*(C6+2.*C7)
      SMAT(4,6) = 2.*A1+PR*C10
      SMAT(4,7) = PR*0.5*C3
      SMAT(4,10)= -1.5*A2+PR*(-1.5*C1-0.5*C3)
      SMAT(4,11)= PR*(-C6)
      SMAT(4,12)= A1+PR*(C9)
C
      SMAT(5,1) = -1.5*C2-0.5*C3
      SMAT(5,2) = C7
      SMAT(5,3) = -C10
      SMAT(5,4) = PR*(1.5*A2) +1.5*C1+1.5*C2+0.5*C3
      SMAT(5,5) = C6+2.*C7
      SMAT(5,6) = PR*2.*A1+C10
      SMAT(5,7) = 0.5*C3
```

```
      SMAT(5,10)= PR*(-1.5*A2) +(-1.5*C1-0.5*C3)
      SMAT(5,11)= -C6
      SMAT(5,12)= PR*A1+C9
C
      SMAT(6,1) = DXY*(-0.5*C5)
      SMAT(6,3) = DXY*(-C12)
      SMAT(6,4) = DXY*(3.*C4+0.5*C5)
      SMAT(6,5) = DXY*C8
      SMAT(6,6) = DXY*(4.*C11+C12)
      SMAT(6,7) = DXY*0.5*C5
      SMAT(6,10)= DXY*(-3.*C4-0.5*C5)
      SMAT(6,11)= DXY*(-C8)
      SMAT(6,12)= DXY*2.*C11
C
      SMAT(7,1) = -1.5*A2 +PR*(-1.5*C1-0.5*C3)
      SMAT(7,2) = PR*C6
      SMAT(7,3) = -A1-PR*C9
      SMAT(7,4) = PR*0.5*C3
      SMAT(7,6) = -PR*C9
      SMAT(7,7) = 1.5*A2 +PR*(1.5*C1+1.5*C2+0.5*C3)
      SMAT(7,8) = PR*(-C6-2.*C7)
      SMAT(7,9) = -2.*A1+PR*(-2.*C9-C10)
      SMAT(7,10)= PR*(-1.5*C2-0.5*C3)
      SMAT(7,11)= -PR*C7
      SMAT(7,12)= PR*C10
C
      SMAT(8,1) = PR*(-1.5*A2) -1.5*C1-0.5*C3
      SMAT(8,2) = C6
      SMAT(8,3) = -PR*A1-C9
      SMAT(8,4) = 0.5*C3
      SMAT(8,6) = -C9
      SMAT(8,7) = PR*1.5*A2 +1.5*C1+1.5*C2+0.5*C3
      SMAT(8,8) = -C6-2.*C7
      SMAT(8,9) = PR*(-2.*A1)-2.*C9-C10
      SMAT(8,10)= -1.5*C2-0.5*C3
      SMAT(8,11)= -C7
      SMAT(8,12)= C10
C
      SMAT(9,1) = DXY*(-3.*C4-0.5*C5)
      SMAT(9,2) = DXY*C8
      SMAT(9,3) = DXY*(-2.*C11)
      SMAT(9,4) = DXY*0.5*C5
      SMAT(9,7) = DXY*(3.*C4+0.5*C5)
      SMAT(9,8) = DXY*(-C8)
      SMAT(9,9) = DXY*(-4.*C11-C12)
      SMAT(9,10)= DXY*(-0.5*C5)
      SMAT(9,12)= DXY*C12
C
      SMAT(10,1) = PR*(-0.5*C3)
      SMAT(10,4) = -1.5*A2 +PR*(-1.5*C1+0.5*C3)
      SMAT(10,5) = PR*C6
      SMAT(10,6) = -A1
```

```
      SMAT(10,7) = PR*(-1.5*C2+0.5*C3)
      SMAT(10,8) = PR*C7
      SMAT(10,9) = -PR*C10
      SMAT(10,10)= 1.5*A2 +PR*(1.5*C1+1.5*C2-0.5*C3)
      SMAT(10,11)= PR*(-C6+2.*C7)
      SMAT(10,12)= -2.*A1 +PR*(-2.*C9+C10)
C
      SMAT(11,1) = -0.5*C3
      SMAT(11,4) = PR*(-1.5*A2)-1.5*C1+0.5*C3
      SMAT(11,5) = C6
      SMAT(11,6) = -PR*A1
      SMAT(11,7) = -1.5*C2+0.5*C3
      SMAT(11,8) = C7
      SMAT(11,9) = -C10
      SMAT(11,10)= PR*1.5*A2+1.5*C1+1.5*C2-0.5*C3
      SMAT(11,11)= -C6+2.*C7
      SMAT(11,12)= PR*(-2.*A1) -2.*C9+C10
C
      SMAT(12,1) = DXY*(-0.5*C5)
      SMAT(12,4) = DXY*(-3.*C4+0.5*C5)
      SMAT(12,5) = DXY*C8
      SMAT(12,6) = DXY*(-2.*C11)
      SMAT(12,7) = DXY*0.5*C5
      SMAT(12,9) = DXY*(-C12)
      SMAT(12,10)= DXY*(3.*C4-0.5*C5)
      SMAT(12,11)= DXY*(-C8)
      SMAT(12,12)= DXY*(-4.*C11+C12)
C
      CALL SMULT(SMAT,D,SMAT,12,12)
C
      RETURN
      END
```

APPENDIX D

USER'S MANUAL FOR FEACONS V

All of the input data are free formatted so that the data are not limited to any specific columns. Adjoining data must be seperated by a blank or a comma. However, a command statement must start at the first column of each line. Inputs to the program are listed below.

Item #	INPUT	Mandatory (M) or Optional (O)
1.	Number of runs	M
2.	Number of x-divisions on slab #1 Number of x-divisions on slab #2 Number of x-divisions on slab #3 Number of y-divisions on slab	M
3.	Degree of skewed angle in degree Normal transverse joint = 90.	M
4.	Thickness of slab (in inches), Elastic modulus of concrete (in ksi), Poisson's ratio of concrete	M
5.	X coordinates of nodes along the X axis (in inches)	M
6.	Y coordinates of nodes along the y axis (in inches)	M
7.	Command LINEAR (for liner subgrade), or NONLINEAR (for nonlinear subgrade)	M
8.	Subgrade modulus in kci (if LINEAR), or Coefficient A, Coefficient B (if NONLINEAR) (The force-deflection relationship is: F = Ad + Bd2, whewe F = force/area in ksi, and d = deflection in inches)	M
9.	Command GAP (if initial gaps are to be read), or NO GAP	M
10.	Skip if NO GAP. Otherwise, input: 1) Number of gaps 2) Node number, Depth of gap in inches (use one line for each node with gap)	M
11.	Density of concrete in pcf (need only to calculate effect of slab wt.)	M
12.	Command TEMPERATURE EFFECT (if effects of temperature differentials are to be considered), or NO TEMPERATURE EFFECT	M

13.	Skip if NO TEMPERATURE EFFECT. Otherwise: Coefficient of thermal expansion (in 1/0F), Temperature at the top of the slab (in oF), Temperature at the bottom of the slab (in oF)	M
14.	Spring coefficient for the edges (in ksi)	M
15.	Linear spring coefficient for the joints (in ksi), Torsional spring coefficient for the joints (in k/in)	M
16.	Linear spring coefficient for the dowel bar Torsional spring coefficient for the dowel bar Amount of slip for the dowel bar	M
17.	Number of load increments to compute the effects of slab weight, Number of load increments to compute the effects of temperature differentials, Number of load increments to compute the effects of applied loads	M
18.	Command CONC FORCE (if concentrated loads are to be read in), or NO CONC FORCE	M
19.	Skip if NO CONC FORCE. Otherwise: Number of Concentrated Forces (on one line) Node number, Magnitude of load in kips (use one line for each node)	M
20.	Command UNIF LOAD (if initial uniform load is to be read in), or NO UNIF LOAD	M
21.	Skip IF NO UNIF LOAD. Otherwise: Number of elements with uniform loads (one one line) Element number, initial Uniform load in ksi (use one line for each element)	M
22.	Command PRINT INITIAL DEFLECTION (if deflections caused by the combined effects of slab weight and temperature differentials are to be printed)	O
23.	If the command PRINT INITIAL DEFLECTION is read in, read in : Total number of sets of nodes to be printed, starting node number, Ending node number, Increment between the nodes. (The last three	O

numbers represent a node set. The next node set follows here if there is more than one node set.)

24. Command PRINT DEFLECTION (if deflections caused by applied loads are to be printed) O

25. If PRINT DEFLECTION is read in, read in: O
Total number of sets of nodes to be printed,
Staring node number,
Ending node number,
Increment between the nodes. (This is similar to item 23)

26. Command SAVEi DEFLECTION O
Save deflections of plate bending element to run INPLANE program.
i = 1 to 5 represents number of incremental loadings.

27. If SAVEi DEFLECTION is read in :
Number of sets of nodes
Starting node number
Ending node number
Increment. (Similar to item 23)

28. Command PRINT MAXIMUM DEFLECTIONS, read in : O
(if maximum deflections between specific nodes are to be printed)

29. If PRINT MAXIMUM DEFLECTIONS, read in : O
Number of sets of nodes
Starting node number
Ending node number
Increment. (Similar to item 23)

30. Command PRINT MOMENTS O
(If moments at the nodes are to be printed)

31. If PRINT MOMENTS, read in : O
Number of sets of nodes
Starting node number
Ending node number
Increment. (Similar to item 23)

32. Command PRINT MAXIMUM MOMENTS (If maximum moments between specific nodes are to be printed) O

33. If PRINT MAXIMUM MOMENTS, read in : O
Number of sets of nodes

Starting node number
Ending node number
Increment. (Similar to item 23)

34. Command PRINT STRESSES O
(If stresses at the nodes are to be printed)

35. If PRINT STRESSES, read in : O
Number of sets of nodes
Starting node number
Ending node number
Increment. (Similar to item 23)

36. COMMAND PRINT MAXIMUM STRESSES O
(If maximum stresses between specific nodes are to be printed)

37. If PRINT MAXIMUM STRESSES, then read in : O
Number of sets of nodes
Starting node number
Ending node number
Increment. (Similar to item 23)

38. COMMAND PRINT PRINCIPAL STRESSES O
(If principal stresses are to be printed)

39. If PRINT PRINCIPAL STRESSES, then read in : O
Number of sets of nodes,
DEG,
Starting node number,
Ending node number,
Increment.
(If DEG = 1, angles will be in degrees.
If DEG = 2, angles willbe in radians.)
(The last four numbers represent a node set.
The next node set follows here if there is more than one node set)

40. Command PRINT MAXIMUM PRINCIPAL STRESSES O
(If maximum principal stresses between specific nodes are to be printed)

41. If PRINT MAXIMUM PRINCIPAL STRESSES, then: O
Number of sets of nodes
Starting node number
Ending node number
Increment. (Similar to item 23)

42. Command PRINT LOCAL MOMENTS O
(If local moments between specific nodes are to be printed.)

43.	If PRINT LOCAL MOMENTS, then : Number of sets of nodes Starting node number Ending node number Increment. (Similar to item 23)	O
44.	Command PRINT LOCAL STRESSES (If local stresses between specific nodes are to be printed.)	O
45.	If PRINT LOCAL STRESSES, then : Number of sets of nodes Starting node number Ending node number Increment. (Similar to item 23)	O
46.	Command INCREMENTAL LOADING, then : Repeat Command 18 through 45	O
47.	Command FINISH (This is to mark the end of a set of data. The next set of data in the same formats as items 2 through 47 follows here, if there is more than one run to be made.)	M

APPENDIX E

USER'S MANUAL FOR THE INPLANE II

The program, INPLANE II, analyzes the joint displacements pavement of a one to three slab system in response to concrete shrinkage, thermal expansion or contraction, and subgrade friction effects.

the first step in using INPLANE is the creation of a file containing the nodal displacements due to plate bending. To do this, the program, FEACONS, must be run using the same nodal mesh as for INPLANE.

All of the input for INPLANE is free-formatted and requires only a comma or blank space to seperate input variables. The program will accept any system of units as long as the input is consistent with the system chosen.

The output from INPLANE is as follows :

1. All input except the coordinate data is echoed.
2. The nodal degrees of freedom.
3. The degrees of freedom of the joint nodes in the x-directon.
4. The final x-coordinates of the displaced joint nodes.
5. The nodal stresses.

If no commands are used, then a blank line must still be included. To skip the calculation of shrinkage effects, input 0.0 for the variable ESHR. To skip the thermal effects, input 0.0 for all of the temperature variables.

INPUT DATA FOR PROGRAM INPLANE

1. NRUN	number of runs to be made.
2. TITLE	descriptive title of each run. if no title is desired must be a blank line.
3. EC,POI,ESHR	elastic modulus, poisson's ratio, shrinkage strain.
4. NSLAB,ANGLE	number of slabs in system. skewed angle in degree.
5. NYC	number of nodes in Y-direction
6. NXC(I)	number of nodes in X-direction I=1.NSLB
7. YCRD(I)	Y-coordinate, I=1, NYC
8. XCRD (I,J)	X-coordinate, I=1, NSLB J=1, NXC(I) each slab must start a new line.
9. GAP (I)	joint spacing, I=1, NSLB-1
10.NVD	number of voids.
11.KVOID (I)	elements with voids underneath I=1, NVD skip if NVD=0.
12.PHI, SGSTF	coefficient of friction, subgrade stiffness.
13.TO,TT,TM,TB -THK, ALPHA	refwewnce temperature, temperatures at top, middle, and bottom of slab, respectively

	slab thickness, coefficient of thermal expansion.
14.READ. DAT	read in the displacement data file (WDISP.DAT) produced by FEACONS.
15.SKIP	end of program.
16.NO PRINT	suppress nodal DOF output.
17.NO STRESS	skip calculation of nodal stresses.
18.BLANK LINE	end of data for this run.

APPENDIX F

A LISTING OF THE FEACONS V PROGRAM

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                      C
C    THIS PROGRAM IS REVISED FOR GOULD COMPUTER WITH    C
C    UNIX SYSTEM BY Byung Wan  Jo   on Fall , 1986.     C
C                                                      C
C    1. REAL TO IMPLICIT DOUBLE PRECISION.              C
C    2. DSQRT,DATAN TO SQRT,ATAN                        C
C    3. REWIND 3,4,7                                    C
C    4. PARALLELOGRAM PLATE BENDING ELEMENT             C
C    5. SAVE STRUCTURAL DISPLACEMENTS DATA              C
C       FOR INPLANE ANALYSIS PROGRAM. file name = DATA  C
C    6. LOOPINGS FOR THE INTERMEDIATE LOADINGS.         C
C                                                      C
C             i ) new stiffness matrix.                C
C             ii) new stress (force transformation) matrC
C             iii)new equivalent load matrix.          C
C                                                      C
C                     * LIMIT OF THIS PROGRAM          C
C                                                      C
C                       # OF X DIVISION : 40           C
C                       # OF Y DIVISION : 12           C
C                                                      C
C                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION MDX(1677),NODSTR(1677),LMON(1677),
     +         LODSTR(1677)
         DIMENSION DEPS(559),BF(1677),BK1(75465),
     +    SDELTA(1677),EDELTA(12),SK(12,12),
     +    THETA(1677),BF1(1677),SDELT1(1677),
     +    PRNSTR(1677),SMAT(12,12),ELMON(12),FMON(1677),
     +    BK(75465),TBF(1677),SDELT2(1677),BF2(1677),
     +    XCOOR(559),YCOOR(559),COORX(43),COORY(13)
         DIMENSION IND1(480),IND3(480)
         INTEGER BW,EN,DUMIND(12)
C        CHARACTER*12 FILE1,FILE2
       CHARACTER*80 COMAND
C
         COMMON /CHAR/ COMAND
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
         COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                NOE,NDISP,NNOD,BW,NBK,IRUN
C        WRITE(*,1001)
C1001    FORMAT('Name of input file?')
C        READ(*,1000) FILE1
C        WRITE(*,1002)
C1002    FORMAT('Name of output file?')
C        READ(*,1000) FILE2
C        OPEN(3,FILE=FILE1,STATUS='OLD')
C        OPEN(4,FILE=FILE2,STATUS='NEW')
C
```

```
        OPEN (UNIT=3,FILE='INPUT',STATUS='OLD')
        OPEN (UNIT=4,FILE='OUTPUT',STATUS='NEW')
        REWIND 3
        REWIND 4
C-----------------------------------------------
C:  READ IN THE NUMBER OF RUNS TO BE MADE.     :
C-----------------------------------------------
        READ (3,*) NRUN
        WRITE (4,1010) NRUN
 1010   FORMAT (' NUMBER OF RUNS = ',I2)
        DO 9000 IRUN = 1,NRUN
        WRITE (4,1020) IRUN
 1020    FORMAT (' RUN NUMBER = ',I2)
C--------------------------------------------------------
C:  READ IN THE NUMBER OF DIVISIONS ON THE X AXIS FOR SLAB
C:  1, 2, & 3, AND THE NUMBER OF DIVISIONS ON THE Y AXIS.
C--------------------------------------------------------
        READ (3,*) NXDIV1,NXDIV2,NXDIV3,NYDIV
        WRITE (4,1030) NXDIV1,NXDIV2,NXDIV3,NYDIV
 1030   FORMAT (' # OF DIVISIONS ON THE X AXIS FOR SLAB
     +  AND # OF DIVISIONS ON THE Y AXIS'/,3(I6),30X,I6)
C
C-------------------------------------
C:  READ IN THE SKEWED ANGLE OF JOINT.
C-------------------------------------
       READ(3,*) ANGLE
     WRITE(4,1032) ANGLE
 1032  FORMAT(/,'  DEGREE OF SKEWED ANGLE : ',F7.2)
C---------------------------
C:  CHANGE ANGLE TO RADIAN. |
C---------------------------
     PI=4.0*ATAN(1.0)
     ANG=ANGLE*PI/180.
C
C--------------------------------------------------------
C:  CALCULATE THE TOTAL  NUMBER OF ELEMENT (NOE), BANDWI
C:  (BW), TOTAL NUMBER OF DEGREES OF FREEDOM (NDISP), TO
C:  NUMBER OF NODES (NNOD), AND THE SIZE OF THE STRUCTUR
C:  STIFFNESS MATRIX (NBK).
C--------------------------------------------------------
        NXDIV = NXDIV1 + NXDIV2 + NXDIV3
        NXNOD = NXDIV + 3
        NYNOD = NYDIV + 1
        NOE = NXDIV * NYDIV
        BW = 3 * (NYNOD + 2)
        NNOD = NXNOD * NYNOD
        NDISP = NXNOD * NYNOD *3
        NBK = BW * NDISP
C----------------------------
C   CHECK PROGRAM CAPACITY.  :
C----------------------------
      IF(NXDIV.GT.40.OR.NYDIV.GT.12) THEN
```

```
      WRITE(4,3000)
      STOP
      END IF
C-----------------------------------------------------
C:  READ IN THE OTHER STRUCTURAL DATA AND INITIALIZE  :
C:  THE STRUCTURAL MESH.                              :
C-----------------------------------------------------
        CALL INDEXG (IND1,IND3,XCOOR,YCOOR,COORX,COORY)
C-----------------------------------------------------
C:  READ IN THE COMMAND LINEAR OR NONLINEAR TO SPECIFY  :
C:  LINEAR OR NON-LINEAR SUBGRADE BEHAVIOR.             :
C-------------------------------------------------------
        READ (3,1000) COMAND(1:9)
        WRITE (4,*) COMAND(1:9)
        IF (COMAND(1:6).EQ.'LINEAR') GO TO 100
C-------------------------------------------------------
C:  IF NONLINEAR SUBGRADE BEHAVIOR IS CONSIDERED, WHERE :
C:  FORCE = A * (DEF) + B * (DEF) ** 2,                 :
C:  READ IN THE CONSTANTS A & B                         :
C-------------------------------------------------------
        READ (3,*) A,B
        WRITE (4,1035) A,B
 1035    FORMAT (' A          B'/,2F10.4)
        LFACT = 2
        GO TO 120
  100     LFACT = 1
        READ (3,*) SMOD
        WRITE (4,1040) SMOD
 1040    FORMAT(' SUBGRADE MODULUS'/,F10.4)
C------------------------------------
C:  READ IN COMMAND GAP OR NOGAP.     :
C------------------------------------
  120     READ (3,1000) COMAND (1:40)
        WRITE (4,*) COMAND (1:40)
C---------------------------------------------------
C:  INITIALIZE THE MATRIX STORING THE INITIAL       :
C:  DEPTHS OF GAPS [DEPS]                           :
C---------------------------------------------------
        CALL ZERO (DEPS,NNOD,1)
        IF (COMAND(1:6).EQ.'NO GAP') GO TO 200
        CALL GAP (DEPS,NNOD)
  200   CONTINUE
C------------------------------------------------------
C| READ IN THE DENSITY OF CONCRETE & CALCULATE THE FORCE
C| BF1 DUE TO THE WEIGHT OF THE SLAB
C------------------------------------------------------
       READ(3,*) DENSTY
       WRITE (4,1042) DENSTY
 1042  FORMAT(' DENSITY OF CONCRETE IN PCF',/,F10.2)
       CALL ZERO(BF1,NDISP,1)
         IF (DENSTY .EQ. 0.) GO TO 370
       CALL SLABWT(BF1,IND1,IND3,XCOOR,YCOOR,DENSTY)
```

```
C-----------------------------------------------------------
C:  READ IN TEMPERATURE EFFECT OR NO TEMPERATURE EFFECT  :
C-----------------------------------------------------------
  370    READ (3,1000) COMAND(1:40)
         WRITE (4,*) COMAND(1:40)
         LTEMP = 0
         CALL ZERO(BF2,NDISP,1)
         IF (COMAND(1:9).EQ.'NO TEMPER') GO TO 390
         CALL TEMPER (BF2,IND1,IND3,XCOOR,YCOOR,BB)
         LTEMP = 1
C----------------------------------------------------------
C|    READ IN THE SPRING COEFFICIENT FOR THE EDGES        |
C----------------------------------------------------------
  390   READ(3,*) CONST1
        WRITE(4,1045) CONST1
 1045   FORMAT(' SPRING COEFFICIENT AT THE EDGE',/,F10.2)
C---------------------------------------------------------
C  READ IN THE LINEAR AND TORSIONAL SPRINGS AT THE JOINT
C---------------------------------------------------------
        READ (3,*) CONST2,CONST3
        WRITE (4,1050) CONST2,CONST3
 1050 FORMAT ('LINEAR & TORSIONAL SPRING  AT THE JOINT',
     +   /,F10.2,2X,F10.2)
C---------------------------------------------------------
CREAD IN THE LINEAR AND TORSIONAL SPRINGS (DWST1 &DWST2)
CSLIP FOR THE DOWEL JOINTS
C---------------------------------------------------------
        READ(3,*)DWST1,DWST2,SLIP
        WRITE(4,1055)DWST1,DWST2,SLIP
 1055   FORMAT(' LINEAR & TORSIONAL SPRING COEFFICIENTS ,
     +    /,F10.2,2X,F10.2,/,'AMOUNT OF SLIP = ',F10.2)
C---------------------------------------------------------
C  READ IN THE NUMBER OF LOAD INCREMENTS TO BE USED
C---------------------------------------------------------
        READ(3,*) NN1,NN2,NN3
        WRITE(4,1060)NN1,NN2,NN3
 1060   FORMAT(' NUMBER OF LOAD INCREMENTS FOR SLAB WEIGHT,
     +    ,' EFFECT AND APPLIED LOADS.',/,3I10)
C---------------------------------------------------------
C  INITIALIZE THE STRUCTURE STIFFNESS MATRIX BK1, THE BW
C  STIFFNESS MATRIX IS STORED IN A ONE-DIMENSIONAL ARRAY
C---------------------------------------------------------
        CALL ZERO (BK1,NBK,1)
C---------------------------------------------------------
C  SET UP THE STRUCTURAL STIFFNESS MATRIX [BK1] BY
C  INSERTING THE ELEMENTAL STIFFNESS MATRIX [SK] INTO THE
C  PROPER POSITIONS IN [BK1]|
C---------------------------------------------------------
        DO 500 EN=1,NOE
          CALL ZERO(SK,12,12)
C---------------------------------------------------------
C  CALCULATE NODE # OF LOWER LEFT NODE, IND11 & OF LOWER
```

```
C  NODE IND33
C--------------------------------------------------------
          IND11=IND1(EN)
          IND33=IND3(EN)
          ELEN=XCOOR(IND33)-XCOOR(IND11)
          EWID=YCOOR(IND11+1) - YCOOR(IND11)
C--------------------------------------------------------
C  OBTAIN ELEMENT STIFFNESS MATRIX  [SK]
C--------------------------------------------------------
          CALL KMAT (SK,ELEN,EWID)
C--------------------------------------------------------
C  INSERT IT INTO THE GLOBAL STIFFNESS MATRIX [BK1]
C--------------------------------------------------------
          CALL INSERT (BK1,SK,IND11,IND33)
  500   CONTINUE
        IF (CONST2 .NE. 0. .OR. CONST3 .NE. 0.) THEN
C--------------------------------------------------------
C  ADJUST [BK1] FOR JOINT CONDITIONS
C--------------------------------------------------------
          CALL JOINT (BK1,CONST2,CONST3,YCOOR)
        ENDIF
C--------------------------------------------------------
C|    CALCULATE THE INITIAL POSITIONS OF THE SLAB DUE TO
C     ITS OWN WEIGHT
C--------------------------------------------------------
      CALL ZERO(SDELTA,NDISP,1)
      CALL CALDEL(BK1,SDELTA,BF1,DEPS,IND1,IND3,NN1,LFACT,
     +    BK,TBF,DELTA,DWST1,DWST2,SLIP)
        IF (CONST1 .NE. 0.) THEN
C--------------------------------------------------------
C|    ADJUST [BK1] FOR EDGE CONDITIONS
C--------------------------------------------------------
          CALL EDGE(BK1,CONST1,XCOOR)
        ENDIF
C---------------------------------------------------------
C  CALCULATE THE TOTAL INITIAL DISPLACEMENTS [SDELTA] AND
C  THEM
C  EQUAL TO [SDELT1]
C---------------------------------------------------------
      CALL CALDEL(BK1,SDELTA,BF2,DEPS,IND1,IND3,NN2,LFACT,
     +    YCOOR,BK,TBF,DELTA,DWST1,DWST2,SLIP)
        DO 517 I=1,NDISP
  517     SDELT1(I)=SDELTA(I)
C--------------------------------------
C:   INITIALIZE THE FORCE VECTOR [BF].  :
C:   LOOPING FOR THE INTERMEDIATE LOAD. :
C--------------------------------------
       IDW = 0
 5000    CALL ZERO (BF,NDISP,1)
         LLOAD=0
C----------------------------------------------
C:  READ IN COMMAND CONC FORCE OR NO CONC FORCE   :
```

```
C---------------------------------------------------
        READ (3,1000) COMAND(1:40)
        WRITE (4,*) COMAND(1:40)
        IF (COMAND(1:7).EQ.'NO CONC') GO TO 320
C-----------------------------------------------------
C:  READ IN THE CONCENTRATED LOADS AND INSERT THEM  :
C:  INTO THE FORCE VECTOR BF.                       :
C-----------------------------------------------------
        CALL FORCEC (BF,NDISP)
       LLOAD=1
C-------------------------------------------
C:  READ IN UNIF LOAD OR NO UNIF LOAD      :
C-------------------------------------------
  320   READ (3,1000) COMAND(1:40)
        WRITE(4,*) COMAND(1:40)
        IF(COMAND(1:7).EQ.'NO UNIF') GO TO 350
        CALL FORCEU (BF,IND1,IND3,XCOOR,YCOOR)
        LLOAD=1
  350  CONTINUE
C--------------------------------------------------------
C  CALCULATE THE   TOTAL   NODAL DISPLACEMENTS [SDELTA]
C--------------------------------------------------------
       IF (LLOAD .NE. 0) THEN
         CALL CALDEL (BK1,SDELTA,BF,DEPS,IND1,IND3,NN3,
     +     YCOOR,BK,TBF,DELTA,DWST1,DWST2,SLIP)
       ENDIF
C--------------------------------------------------------
C|CALCULATE THE DIFFERENCE BETWEEN TOTAL AND INITIAL
C DEFLECTION TO OBTAIN THE MEASURED DEFLECTIONS
C--------------------------------------------------------
       CALL SUBTRC(SDELTA,SDELT1,SDELT2,NDISP,1)
       READ(3,1000) COMAND(1:40)
       WRITE(4,*) COMAND(1:40)
       IF (COMAND(1:13) .EQ. 'PRINT INITIAL') THEN
         WRITE(4,1090)
 1090  FORMAT( ' ',/,'INITIAL DEFLECTIONS DUE TO SLAB ',
     +    ' AND TEMPERATURE EFFECTS:')
         CALL DELWRT(SDELT1)
         READ (3,1000) COMAND(1:40)
         WRITE(4,*) COMAND(1:40)
       END IF
       IF (COMAND(1:10) .EQ. 'PRINT DEFL') THEN
         WRITE(4,1100)
 1100 FORMAT(' ',/,' DEFLECTIONS CAUSED BY APPLIED LOADS')
         CALL DELWRT (SDELT2)
         READ(3,1000) COMAND(1:40)
         WRITE(4,*)COMAND(1:40)
      END IF
CC
C---------------------------------------------------------
C    SAVE PLATE BENDING DISPLACEMENTS OF EACH INTERMEDIATE
C    LOADINGS FOR THE INPLANE ANALYSIS.
```

```
C-----------------------------------------------------------
        IF (COMAND(1:5) .EQ. 'SAVE ') THEN
         OPEN (UNIT=7,FILE='DATA',STATUS='NEW')
         REWIND 7
          WRITE(7,1105) COMAND(1:5)
          CALL INPWRT (SDELTA)
          READ(3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
       END IF
CC
        IF (COMAND(1:5) .EQ. 'SAVE1') THEN
         OPEN (UNIT=7,FILE='DATA1',STATUS='NEW')
         REWIND 7
          WRITE(7,1105) COMAND(1:5)
          CALL INPWRT (SDELTA)
          READ(3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
       END IF
CC
        IF (COMAND(1:5) .EQ. 'SAVE2') THEN
         OPEN (UNIT=7,FILE='DATA2',STATUS='NEW')
         REWIND 7
          WRITE(7,1105) COMAND(1:5)
          CALL INPWRT (SDELTA)
          READ(3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
       END IF
CC
        IF (COMAND(1:5) .EQ. 'SAVE3') THEN
         OPEN (UNIT=7,FILE='DATA3',STATUS='NEW')
         REWIND 7
          WRITE(7,1105) COMAND(1:5)
          CALL INPWRT (SDELTA)
          READ(3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
       END IF
CC
        IF (COMAND(1:5) .EQ. 'SAVE4') THEN
         OPEN (UNIT=7,FILE='DATA4',STATUS='NEW')
         REWIND 7
          WRITE(7,1105) COMAND(1:5)
          CALL INPWRT (SDELTA)
          READ(3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
       END IF
CC
        IF (COMAND(1:5) .EQ. 'SAVE5') THEN
         OPEN (UNIT=7,FILE='DATA5',STATUS='NEW')
         REWIND 7
          WRITE(7,1105) COMAND(1:5)
          CALL INPWRT (SDELTA)
          READ(3,1000) COMAND(1:40)
```

```
          WRITE(4,*)COMAND(1:40)
       END IF
 1105  FORMAT(5X,A5,2X,'TOTAL DEFLECTIONS FOR INPLANE .')
CC
   IF (COMAND(1:24) .EQ. 'PRINT MAXIMUM DEFLECTION') THEN
          CALL MAXMUM (SDELTA)
          READ (3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
        END IF
C-----------------------------------------------------------
C FOR EACH ELEMENT;
C   (1) EXTRACT THE 12 X 1 ELEMENTAL DISPLACEMENT VECTOR
C   {EDELTA} FROM THE NDISP X 1 STRUCTURAL DISPLACEMENT
C    VECTOR {DELTA}.
C (2) EVALUATE THE 12 X 12 ELEMENTAL STRESS MATRIX [SMAT].
C (3) CALCULATE THE 12 X 1 ELEMENTAL STRESS-COUPLE VECTOR
C      {ELMOM} BY {ELMOM} = [SMAT]*{EDELTA}.
C (4) INSERT {ELMOM} INTO THE NDISP X 1 STRUCTURAL STRESS-
C     MATRIX [FMOM].
C-----------------------------------------------------------
        CALL ZERO (MDX,NDISP,1)
        CALL ZERO (FMON,NDISP,1)
        DO 600 EN=1,NOE
C-----------------------------------------------------------
C    CALCULATE NODE# OF LOWER LEFT NODE,IND11, & OF LOWER
C    RIGHT NODE, IND33
C-----------------------------------------------------------
          IND11=IND1(EN)
          IND33=IND3(EN)
          ELEN=XCOOR(IND33)-XCOOR(IND11)
          EWID=YCOOR(IND11+1)-YCOOR(IND11)
C----------------------------------------------------------
C    DETERMINE THE STRUCTURAL COORDINATES OF THE ELEMENT
C    & STORE THEM IN [DUMIND]
C----------------------------------------------------------
          DUMIND(1)=(IND11-1)*3+1
          DUMIND(7)=(IND33-1)*3+1
          DO 520 I=1,5
            DUMIND(I+1)=DUMIND(1)+I
  520       DUMIND(I+7)=DUMIND(7)+I
          CALL XTRACT(SDELTA,EDELTA,DUMIND,NDISP)
          CALL STRMAT(SMAT,ELEN,EWID,E,T,PR,ANGLE,ANG)
          CALL MULT(SMAT,EDELTA,ELMON,12,12,1)
C---------------------------------------------------
C    ADD THE THERMAL STRESS-COUPLES IF TEMPERATURE
C    EFFECTS ARE CONSIDERED
C---------------------------------------------------
          IF (LTEMP .EQ. 1) THEN
            CALL TEMPFM(ELMON,BB)
          END IF
          CALL BENSET(FMON,ELMON,DUMIND,NDISP,MDX)
  600   CONTINUE
```

```
C-----------------------------------------------------------
C|  CALCULATE THE AVERAGE STRESS-COUPLES FOR EACH NODE. |
C-----------------------------------------------------------
        CALL DIVIDE (MDX,FMON,NDISP)
        IF (COMAND(1:13) .EQ. 'PRINT MOMENTS') THEN
          CALL MOMWRT (FMON)
          READ(3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
        END IF
      IF (COMAND(1:21) .EQ. 'PRINT MAXIMUM MOMENTS') THEN
          CALL MAXMUM (FMON)
          READ (3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
        END IF
C------------------------------------------------------------
C|  CALCULATE THE AVERAGE STRESS-INTENSITIES FOR EACH NODE
C------------------------------------------------------------
        CALL STRESS (FMON,NODSTR)
        IF (COMAND(1:14) .EQ. 'PRINT STRESSES') THEN
          CALL MOMWRT (NODSTR)
          READ(3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
        END IF
    IF (COMAND(1:22) .EQ. 'PRINT MAXIMUM STRESSES') THEN
          CALL MAXMUM (NODSTR)
          READ (3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
        END IF
C---------------------------------------------------
C|  CALCULATE THE PRINCIPAL STRESES-INTENSITIES
C---------------------------------------------------
        CALL PRSTRS (NODSTR,PRNSTR,THETA)
   IF (COMAND(1:24) .EQ. 'PRINT PRINCIPAL STRESSES') THEN
          CALL PSWRT (PRNSTR,THETA)
          READ(3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
        END IF
IF (COMAND(1:32) .EQ. 'PRINT MAXIMUM PRIN STRESSES') THEN
          CALL MAXMUM (PRNSTR)
          READ (3,1000) COMAND(1:40)
          WRITE(4,*)COMAND(1:40)
        END IF
C
C------------------------------------------------------
C   CALCULATE LOCAL STRESSES AND MOMENTS ALONG THE
C   FACE OF SKEWED JOINT USING THE MOHR'S CIRCLE.
C------------------------------------------------------
C
      IF(COMAND(1:19).EQ.'PRINT LOCAL MOMENTS') THEN
        CALL LOCALF(FMON,LMON)
        IF(ANGLE.EQ.90.) CALL MOMWRT(FMON)
        CALL MOMWRT(LMON)
```

```
        READ(3,1000) COMAND(1:40)
        WRITE(4,*) COMAND(1:40)
        END IF
C
      IF(COMAND(1:20).EQ.'PRINT LOCAL STRESSES') THEN
        CALL LOCALF(NODSTR,LODSTR)
        IF(ANGLE.EQ.90.) CALL MOMWRT(NODSTR)
        CALL MOMWRT(LODSTR)
        READ(3,1000) COMAND(1:40)
        WRITE(4,*) COMAND(1:40)
      END IF
C
C
      IF (COMAND(1:9) .EQ. 'INCREMENT') THEN
           IDW = IDW + 1
         WRITE(4,5100) IDW
         GO TO 5000
         END IF
C
        IF (COMAND(1:6) .NE. 'FINISH') THEN
          WRITE(4,2010)
          COMAND = 'FINISH'
        END IF
        WRITE(4,2000) COMAND(1:6)
 9000 CONTINUE
 1000   FORMAT (A)
 2000   FORMAT (//' ',12('*')/' ','** ',A6,' **'/' '//)
 2010   FORMAT (//' ','    COMMANDS OUT OF ORDER'//)
 3000   FORMAT (//' ',' NXDIV OR NYDIV .GT. MAXIMUM'//)
 5100   FORMAT (/////' ,70('=')//' ',20X,'OUTPUTS
     -  FOR THE INCREMENTAL LOADING ',I3,//',70('=')//)
        END
C
C
C:--------------------------------------------
C: SUBROUTINE ADD WILL ADD THE N X M MATRIX [A]
C:       AND N X M MATRIX [B] UP:
C:--------------------------------------------
C
      SUBROUTINE ADD (A,B,N,M)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION A(N,M),B(N,M)
      DO 10 I=1,N
      DO 10 J=1,M
  10    A(I,J)=A(I,J)+B(I,J)
      RETURN
      END
C
C
C:----------------------------------------------------
C: SUBROUTINE BENSET WILL INSERT A ELEMENT MOMENT VECTOR
C: OF 12 X 1  :
```

```
C: TO CREATE A STRUCTURE MOMENT VECTOR [A] OF N X1.
C: [MDX] STORES NUMBER OF ADJOINING ELEMENTS AT THE NODES
C:--------------------------------------------------------
C
      SUBROUTINE BENSET(A,B,DUMIND,N,MDX)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION MDX(N)
      DIMENSION A(N),B(12)
      INTEGER DUMIND(12)
        DO 10 I=1,12
           II=DUMIND(I)
           A(II) = A(II) + B(I)
           MDX(II) = MDX(II) + 1
 10     CONTINUE
      RETURN
      END
C
C
      SUBROUTINE CALDEL (BK1,SDELTA,BF,DEPS,IND1,IND3,NN,
     +                  YCOOR,BK,TBF,DELTA,DWST1,DWST2)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION BK1(NBK),BK(NBK),SDELTA(NDISP),BF(NDISP),
     +  DEPS(NNOD),EDELTV(4),TBF(NDISP),DELTA(NDISP),X(4),
     +  EDEPS(4),XCOOR(NNOD),YCOOR(NNOD)
      DIMENSION IND1(NOE),IND3(NOE),INDEX(4)
      INTEGER BW,EN
        COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
        COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                 NOE,NDISP,NNOD,BW,NBK,IRUN
C
      DO 500 INCR=1,NN
C------------------------------------------------------
C    SET [BK] EQUAL TO [BK1] SO THAT [BK1] WILL REMAIN
C------------------------------------------------------
        DO 100 I=1,NBK
  100     BK(I)=BK1(I)
        DO 450 EN=1,NOE
C------------------------------------------------------
C    EXTRACT THE VERTICAL DISPLACEMENTS FOR ELEMENT EN
C    AND STORE IN MATRIX [EDELTV]
C-----------------------------------------------------
          I1=(IND1(EN)-1)*3+1
          EDELTV(1)=SDELTA(I1)
          EDELTV(2)=SDELTA(I1+3)
          I3=(IND3(EN)-1)*3+1
          EDELTV(3)=SDELTA(I3)
          EDELTV(4)=SDELTA(I3+3)
C------------------------------------------------------
C    EXTRACT THE ELEMENTAL GAPS [EDEPS] FROM THE STRUCTURE
C    GAPS [DEPS] FOR ELEMENT EN
C------------------------------------------------------
          J1=IND1(EN)
```

```
          EDEPS(1)=DEPS(J1)
          EDEPS(2)=DEPS(J1+1)
          J3=IND3(EN)
          EDEPS(3)=DEPS(J3)
          EDEPS(4)=DEPS(J3+1)
C-------------------------------------------------------
C CALCULATE THE FINAL GAPS FOR THE ELEMENT AND STORE IN
C [X]
C-------------------------------------------------------
        DO 200 I=1,4
  200     X(I)=EDELTV(I)-EDEPS(I)
C-------------------------------------------------------
C    CALCULATE THE ELEMENTAL SUBGRADE STIFFNESS AND STORE
C    IN  [ESMOD]
C--------------------------------------------------------
        ELEN=XCOOR(IND3(EN))-XCOOR(IND1(EN))
        EWID=YCOOR(IND1(EN)+1)-YCOOR(IND1(EN))
        AREA = ELEN * EWID / 4.
        DO 300 I = 1,4
          IF (X(I) .LT. 0.) THEN
            ESMOD(I)=0.
            ELSE IF (LFACT .EQ. 2) THEN
              ESMOD(I)=(A + 2 * B * X(I)) * AREA
              ELSE
              ESMOD(I) = SMOD * AREA
          END IF
  300   CONTINUE
C--------------------------------------------------------
C    INSERT THE SUBGRADE STIFFNESSES [ESMOD] INTO THE
C    STRUCTURE STIFFNESS MATRIX [BK].
C    THE LOCATIONS WITHIN THE MATRIX [BK] ARE CALCULATED
C    AND  STORED IN [INDEX]
C--------------------------------------------------------
        INDEX(1)=(I1 -1) * BW + 1
        INDEX(2)=INDEX(1)+3*BW
        INDEX(3)=(I3-1)*BW+1
        INDEX(4)=INDEX(3)+3*BW
        DO 400 I=1,4
  400     BK(INDEX(I))=BK(INDEX(I))+ESMOD(I)
  450   CONTINUE
C---------------------------------------------------------
C  ADJUST [BK] FOR DOWEL JOINT STIFFNESSES               |
C---------------------------------------------------------
        IF (DWST1 .NE. 0. .OR. DWST2 .NE. 0.) THEN
          CALL DJOINT(BK,DWST1,DWST2,SLIP,YCOOR,SDELTA)
        ENDIF
C--------------------------------------------------------
C    CALCULATE THE INCREMENTAL LOAD VECTOR [TBF]         |
C--------------------------------------------------------
        FACTOR = 1./NN
        CALL SMULT (BF,FACTOR,TBF,NDISP,1)
C--------------------------------------------------------
```

```
C    CALCULATE THE INCREMENTAL DISPLACEMENTS [DELTA]      |
C-----------------------------------------------------------
        CALL GAUSS (DELTA,BK,TBF)
C----------------------------------------------------------
C    ADD THE INCREMENTAL DISPLACEMENTS [DELTA] TO THE
C    TOTAL DISPLACEMENTS [SDELTA].
C----------------------------------------------------------
        CALL ADD(SDELTA,DELTA,NDISP,1)
  500 CONTINUE
      RETURN
      END
C
C
C------------------------------------------------------
C|  SUBROUTINE DELWRT WILL PRINT OUT THE STRUCTURAL
C   DISPLACEMENTS IN TABULAR FORM.
C------------------------------------------------------
C
        SUBROUTINE DELWRT (SDELTA)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION SDELTA(NDISP),NODES(48,3)
        INTEGER BW
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
  COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
     +                 NOE,NDISP,NNOD,BW,NBK,IRUN
C
        WRITE(4,2000) IRUN
C-----------------------------------------
C:  IF ALL THE NODES ARE TO BE PRINTED.  :
C-----------------------------------------
        WRITE(4,2005)
        READ(3,*) NUMINC,((NODES(I,J),J=1,3),I=1,NUMINC)
  IF (NODES(1,1) .EQ. 1 .AND. NODES(1,2) .EQ. NNOD .AND.
     +       NODES(1,3) .EQ. 1) THEN
          NN = 0
          DO 40 I = 1,NXNOD
            DO 30 J = 1,NYNOD
              NN = NN+1
              IW=NN*3-2
              IX=IW+1
              IY=IW+2
              SDELTA(IW)=SDELTA(IW)*25400.
      WRITE(4,2010) NN,SDELTA(IW),SDELTA(IX),SDELTA(IY)
              SDELTA(IW)=SDELTA(IW)/25400.
   30       CONTINUE
            WRITE(4,2020)
   40     CONTINUE
        ELSE
          DO 60 I = 1,NUMINC
            DO 50 J = NODES(I,1),NODES(I,2),NODES(I,3)
              IW=J*3-2
              IX=IW+1
```

```
              IY=IW+2
              SDELTA(IW)=SDELTA(IW)*25400.
       WRITE(4,2010) J,SDELTA(IW),SDELTA(IX),SDELTA(IY)
              SDELTA(IW)=SDELTA(IW)/25400.
   50       CONTINUE
            WRITE(4,2020)
   60     CONTINUE
        END IF
 2000   FORMAT ('1',79('=')/,'DEFLECTIONS FOR CASE',I2/
     +          ' ',79('=')////
     +          ',14X,'THETAX',11X,'THETAY'/59('-')//)
 2005  FORMAT ('1',10X,'W IS IN E-3 MILLIMETERS')
 2010   FORMAT (' ',I4,3(4X,D14.8))
 2020   FORMAT (' ')
        RETURN
        END
C
C
C
      SUBROUTINE DIVIDE(MDX,A,N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION MDX(N)
      DIMENSION A(N)
C
      DO 10 I=1,N
      A(I)=A(I)/MDX(I)
 10   CONTINUE
      RETURN
      END
C
C
C-------------------------------------------------------
C SUBROUTINE DJOINT WILL INSERT THE VERTICAL AND TORSIONAL
C SPRINGS AT THE JOINTS TO THE STRUCTURAL STIFFNESS MATRIX
C [BK1] FOR DOWEL JOINTS
C--------------------------------------------------------
C
    SUBROUTINE DJOINT (BK1,DWST1,DWST2,SLIP,YCOOR,SDELTA)
    IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    DIMENSION BK1(NBK),SPRING(2),SDELTA(NDISP),SPR(2)
   +, YCOOR(NNOD),EWID,SLIP
    DIMENSION NS(2),ICONST(2)
      INTEGER BW
        COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
     COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
     +                 NOE,NDISP,NNOD,BW,NBK,IRUN
C
C---------------------------------------------------
C THE SMALLEST NODE # OF JOINT 1 & 2 , NS(1) & NS(2)|
C---------------------------------------------------
      NS(1)=NXDIV1 *NYNOD+1
      NS(2)=(NXDIV1+NXDIV2+1)*NYNOD+1
```

```
      DO 500 I=1,2
C----------------------------------------------------
C          I=1   FOR JOINT #1                        |
C          I=2   FOR JOINT #2                        |
C----------------------------------------------------
        DO 500 J=1,NYDIV
          EWID = YCOOR(NS(I)+J)-YCOOR(NS(I)+J-1)
C--------------------------------------------------------
C THE VERTICAL SPRING CONSTANT,SPRING(1) & THE TORSIONAL
C SPRING CONSTANT, SPRING(2)
C--------------------------------------------------------
          SPRING(1)=DWST1*EWID/2.
          SPRING(2)=DWST2*EWID/2.
C------------------------------------------------------
C   ICONST(1) IS TO BE USED FOR VERTICAL SPRING (K=1)
C   ICONST(2) FOR TORSIONAL SPRING (K=2)
C-----------------------------------------------------
          ICONST(1)=0
          ICONST(2)=2
          DO 500 K=1,2
C--------------------------------------------------------
C          L=1    FOR UPPER NODE OF ELEMENT
C          L=2    FOR LOWER NODE OF ELEMENT
C--------------------------------------------------------
            DO 500 L=1,2
C--------------------------------------------------------
C    CALCULATE THE STRUCTURAL COORDINATE OF THE LEFT NODE,
C    THAT OF THE RIGHT NODE, IR1. CALCULATE THE DIFFERENCE
C    (DIFF) & CALCULATE THE STIFFNESS BASED ON DIFF    |
C---------------------------------------------------------
              IL1=(NS(I)+J+L-3)*3+1
              IR1=IL1 + (NYNOD*3)
              DIFF=ABS(SDELTA(IL1)-SDELTA(IR1))
              IF (DIFF .LT. SLIP) THEN
                SPR(K)=DIFF/SLIP*SPRING(K)
              ELSE
                SPR(K)=SPRING(K)
              ENDIF
C--------------------------------------------------
C    ADD THE SPRING CONSTANTS TO THE APPROPRIATE
C    POSITIONS IN THE STRUCTURE STIFFNESS MATRIX [BK1] |
C--------------------------------------------------------
              IL=IL1+ICONST(K)
              IR=IR1 + ICONST(K)
              ILBK=(IL -1)*BW+1
              IRBK=(IR-1)*BW+1
              BK1(ILBK)=BK1(ILBK)+SPR(K)
              BK1(IRBK)=BK1(IRBK)+SPR(K)
C-------------------------------------------------
C    ADD THE NEGATIVE OF THE SPRING CONSTANTS TO
C    THE APPROPRIATE POSITIONS IN [BK1]
C-------------------------------------------------
```

```
          JRBK=ILBK+NYNOD*3
          BK1(JRBK)=BK1(JRBK)-SPR(K)
500 CONTINUE
    RETURN
    END
```

```
C-----------------------------------------------------------
C|    SUBROUTINE EDGE WILL INSERT THE EDGE SPRINGS TO THE
C|    STRUCTURE STIFFNESS MATRIX [BK1]
C-----------------------------------------------------------
C
      SUBROUTINE EDGE(BK1,CONST1,XCOOR)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION BK1(NBK),XCOOR(NNOD)
      INTEGER BW
      COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
         COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                 NOE,NDISP,NNOD,BW,NBK,IRUN
C
      NXP2=NXDIV+2
      DO 500 I=1,NXP2
C----------------------------------------------------------
C|    CALCULATE ELEMENT LENGTH, ELEN & SPRING CONSTANT,
C|    FOR CONVENIENCE OF PROGRAMMING, THERE IS AN ASSUMED
C     ELEMENT
C|    BETWEEN TWO NODES AT THE JOINT WITH LENGTH OF ZERO.
C----------------------------------------------------------
        L1=(I-1)*NYNOD+1
        L2=L1+NYNOD
        ELEN=XCOOR(L2)-XCOOR(L1)
        SPRING=CONST1*ELEN/2.
C----------------------------------------------------------
C|    CALCULATE THE STRUCTURE COORDINATES FOR THE VERTICAL
C|    DISPLACEMENTS FOR THE LOWER ELEMENT, IL1 & IL2, AND
C|    UPPER ELEMENT, IU1 & IU2
C----------------------------------------------------------
        IL1=(L1-1)*3+1
        IL2=IL1+NYNOD*3
        IU1=I*NYNOD*3-2
        IU2=IU1+NYNOD*3
C-----------------------------------------------------------
C|    ADD THE SPRING CONSTANTS TO THE APPROPRIATE POSITIONS
C     IN THE STRUCTURE STIFFNESS MATRIX [BK1]
C-----------------------------------------------------------
        IL1BK=(IL1-1)*BW+1
        IL2BK=(IL2-1)*BW+1
        IU1BK=(IU1-1)*BW+1
        IU2BK=(IU2-1)*BW+1
        BK1(IL1BK)=BK1(IL1BK)+SPRING
        BK1(IL2BK)=BK1(IL2BK)+SPRING
        BK1(IU1BK)=BK1(IU1BK)+SPRING
        BK1(IU2BK)=BK1(IU2BK)+SPRING
  500 CONTINUE
      RETURN
      END
C
C
C--------------------------------------------------------------
```

```
C    SUBROUTINE FORCEC WILL READ IN THE CONCEN. FORCES AND |
C    INSERT THEM INTO THE STRUCTURAL FORCE VECTOR [BF]     |
C-----------------------------------------------------------
C
      SUBROUTINE FORCEC (BF,NDISP)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION BF(NDISP)
C-----------------------------------------------------
C    READ IN THE NUMBER OF CONCENTRATED LOADS          |
C-----------------------------------------------------
      READ(3,*)NUML
      WRITE(4,1010) NUML
 1010 FORMAT(' NUMBER OF CONCENTRATED LOADS TO BE READ,
     +  ' NODE #   CONCENTRATED LOAD')
C-----------------------------------------------------
C    READ IN THE NODE # AND THE MAGNITUDE OF LOAD AND
C    INSERT THE|LOAD INTO THE FORCE VECTOR [BF]
C-----------------------------------------------------
      DO 10 I=1,NUML
        READ (3,*) NODE,ZFORCE
        WRITE(4,1020)NODE,ZFORCE
          NN=NODE*3-2
   10     BF(NN)=BF(NN)+ZFORCE
 1020 FORMAT(I6,F10.4)
      RETURN
      END
C
C
C-------------------------------------------------------
C:  SUBROUTINE FORCEU WILL READ IN THE NO. OF ELEM. WITH :
C:  UNIFORM DISTRIBUTED LOAD, THE NODE #, AND THE MAGN . :
C:  THE DIST.LOAD.  THE 12 ELEMENTAL FORCES    WILL BE   :
C:  CALCULATED AND INSERTED INTO THE STRUC.FORCE VECTOR  :
C--------------------------------------------------------
C
         SUBROUTINE FORCEU (BF,IND1,IND3,XCOOR,YCOOR)
         IMPLICIT DOUBLE PRECISION (A-H,O-Z)
         DIMENSION IND1(NOE),IND3(NOE),BF(NDISP),
     +             YCOOR(NNOD)
         INTEGER EN,BW
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
         COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +    NOE,NDISP,NNOD,BW,NBK,IRUN
C-------------------------------
C:  READ IN THE NUMBER OF LOAD  :
C-------------------------------
         READ (3,*) NUML
         WRITE (4,1010) NUML
C-----------------------------------------
C:  READ IN ELEMENT # AND MAGNITUDE OF LOAD  :
C-----------------------------------------
         DO 10 I=1,NUML
```

```
          READ (3,*) EN,ULOAD
          WRITE (4,1020) EN,ULOAD
C--------------------------------------------------------
C|  DETERMINE THE ELEMENT'S LENGTH, WIDTH, AND NODE INC. |
C--------------------------------------------------------
           ELEN = XCOOR(IND3(EN)) - XCOOR(IND1(EN))
           EWID = YCOOR(IND3(EN)+1) - YCOOR(IND3(EN))
           TLOAD = ULOAD*ELEN*EWID
           II = IND1(EN)*3-2
           JJ = II + 3
           KK = IND3(EN)*3-2
           LL = KK + 3
C--------------------------------------------------------
C|  CALCULATE THE NODAL FORCES INDUCED BY THE UNI. LOAD. |
C--------------------------------------------------------
           F1 = TLOAD/4.0
           F2 = TLOAD*EWID/(24.0*SIN(ANG))
           F3 = TLOAD*ELEN/24.0
C---------------------------------------------------------
C|  INSERT THE FORCE OR COUPLE INTO THE PROPER POSITION  |
C|  IN {BF}.                                             |
C---------------------------------------------------------
           BF(II)   = BF(II)+F1
           BF(II+1) = BF(II+1)-F2
           BF(II+2) = BF(II+2)+F3
           BF(JJ)   = BF(JJ)+F1
           BF(JJ+1) = BF(JJ+1)+F2
           BF(JJ+2) = BF(JJ+2)+F3
           BF(KK)   = BF(KK)+F1
           BF(KK+1) = BF(KK+1)-F2
           BF(KK+2) = BF(KK+2)-F3
           BF(LL)   = BF(LL)+F1
           BF(LL+1) = BF(LL+1)+F2
           BF(LL+2) = BF(LL+2)-F3
   10   CONTINUE
 1010   FORMAT (' NUMBER OF ELEMENTS WITH UNI. DIST. LOAD ='
     + ,I4,/,' ELEMENT #   UNIFORM DISTRIBUTED LOAD')
 1020   FORMAT (I6,5X,F10.4)
      RETURN
      END
C
C
C:---------------------------------------------------------
C: SUBROUTINE GAP WILL BE USED TO INPUT LOCATION AND MAG.:
C: THE GAPS.  THE GAPS ARE STORED IN THE ARRAY [DEPS].   :
C:---------------------------------------------------------
C
      SUBROUTINE GAP(DEPS,NNOD)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION DEPS(NNOD)
C
      READ(3,*)NGAP
```

```
      WRITE(4,1010)NGAP
      DO 200 I=1,NGAP
      READ(3,*)NOGAP,ZDEPS
      WRITE (4,1020) NOGAP,ZDEPS
      DEPS(NOGAP) = ZDEPS
  200 CONTINUE
 1010 FORMAT (' NUMBER OF GAPS TO BE READ IN =',I4,/,
     + ' NODE #   DEPTH OF GAP')
 1020 FORMAT (I4,3X,F7.5)
      RETURN
      END
C
C
C-------------------------------------------------------------
C|   SUBROUTINE GAUSS WILL SOLVE A MATRIX EQUATION OF THE
C|   FORM [A] * {X} = {Y} WHERE [A] IS A M X M MATRIX WHICH
C|   HAS A BANDWIDTH OF 2*BW - 1. THE SOLUTION IS A STANDARD
C|   GAUSS ELIMINATION FOR A SYMMETRICALLY-BANDED MATRIX.
C|   THE MATRIX [C] IS USED TO STORE THE UPPER-TRIANGULAR
C:   NON-ZERO VALUES OF THE MATRIX [A] IN A ONE-DIMENSIONAL
C:   ARRAY.
C-------------------------------------------------------------
C
      SUBROUTINE GAUSS (X,C,Y)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(NDISP),C(NBK),Y(NDISP)
      INTEGER BW
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
         COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                 NOE,NDISP,NNOD,BW,NBK,IRUN
C
C-----------------------
C|   FORWARD ELIMINATION |
C-----------------------
        M = BW-1
        NM1 = NDISP-1
        DO 30 J = 1,NM1
           JM = J+M
           IF (JM .GT. NDISP) JM = NDISP
           JP1 = J+1
           MM = M+1
           IF (M+J .GT. NDISP) MM = NDISP-J+1
           DO 20 K = JP1,JM
              KK = K-JP1+2
C------------------------------------------------------------
C:  CONVERT THE TWO-DIMENSIONAL SUBSCRIPT (J,KK) INTO THE
C:  ONE-DIMENSIONAL SUBSCRIPT (JKK), AND (J,1) INTO (J1).
C------------------------------------------------------------
              JKK = (J-1) * BW + KK
              J1 = (J-1) * BW + 1
              COEFF = -C(JKK) / C(J1)
              Y(K) = Y(K)+COEFF*Y(J)
```

```
              MM = MM-1
              DO 10 I = 1,MM
               II = I+K-J
               KI = (K-1) * BW + I
               JII = (J-1) * BW + II
               C(KI) = C(KI) + COEFF * C(JII)
   10         CONTINUE
   20      CONTINUE
   30   CONTINUE
C-------------------------
C|  BACKWARD SUBSTITUTION |
C-------------------------
        NDISP1 = (NDISP - 1) * BW + 1
        X(NDISP) = Y(NDISP)/C(NDISP1)
        NM = 0
        DO 70 KK = 2,NDISP
           K = NDISP-KK+1
           IF(KK .GT. M) GOTO 40
           NM = NM+1
           GOTO 50
   40   NM = M
   50   X(K) = Y(K)
        N1 = 1
        N2 = K
        DO 60 II = 1,NM
           N1 = N1+1
           N2 = N2+1
           KN1 = (K-1) * BW + N1
           X(K) = X(K)-C(KN1)*X(N2)
   60   CONTINUE
        K1 = (K-1) * BW + 1
        X(K) = X(K)/C(K1)
   70   CONTINUE
        RETURN
        END
C
C
C----------------------------------------------------
C:  SUBROUTINE INDEXG WILL READ IN THE THICKNESS, E, PR
C:  OF THE SLAB, AND THE X AND Y COORD. ALONG THE
C:  X AND Y AXES.  THE SUBROUTINE WILL GENERATE  [IND1]
C:  [IND3], [XCOOR] AND [YCOOR] AND DEFINED BELOW.
C|
C|  VARIABLES:  DEFINITION:
C|  ------------------------------------------------
C|  T           SLAB THICKNESS
C|  E           MODULOUS OF ELASTICITY
C|  PR          POISSON'S RATIO
C|  NXNOD       NO. OF NODES ON THE X-AXIS
C|  NYNOD       NO. OF NODES ON THE Y-AXIS
C|  NOE         TOTAL NO. OF ELEMENTS
C|  NNOD        TOTAL NO. OF NODES
```

```
C|   NXDIV        NO. OF ELEMENTS ON THE X-AXIS
C|   NYDIV        NO. OF ELEMENTS ON THE Y-AXIS
C|
C|   MATRIX:      DEFINITION:
C|   ----------------------------------------------------
C:   [IND1] MATRIX STORING THE STRUCTURAL NODE # OF THE LOWER
C:                LEFT NODE OF ELEMENTS
C:   [IND3] MATRIX STORING THE STRUCTURAL NODE # OF THE LOWER
C:                RIGHT NODE OF ELEMENTS.
C:   [XCOOR]     MATRIX STORING THE X-COORDINATES OF THE NODES
C:   [YCOOR]     MATRIX STORING THE Y-COORDINATES OF THE NODES
C|
C-------------------------------------------------------------
C
       SUBROUTINE INDEXG (IND1, IND3, XCOOR, YCOOR,COORX)
       IMPLICIT DOUBLE PRECISION (A-H,O-Z)
     DIMENSION XCOOR(NNOD),YCOOR(NNOD),COORX(NXNOD),
    +           COORY(NYNOD),IND1(NOE),IND3(NOE)
       INTEGER BW,EN
     COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
    COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
    +                NOE,NDISP,NNOD,BW,NBK,IRUN
C
C-------------------------------------------------------------
C|  READ IN THE SLAB THICKNESS T, MODULUS OF ELASTICITY E, |
C:  AND POISSON'S RATIO PR.                                 :
C-------------------------------------------------------------
       READ(3,*) T,E,PR
       WRITE(4,1010) T,E,PR
C-------------------------------------
C|  READ IN THE X AND Y COORDINATES. |
C-------------------------------------
       READ(3,*) (COORX(I),I=1,NXNOD)
       WRITE(4,1020)
       WRITE(4,1040)(COORX(I),I=1,NXNOD)
       READ(3,*) (COORY(I),I=1,NYNOD)
       WRITE(4,1030)
       WRITE(4,1040)(COORY(I),I=1,NYNOD)
C-------------------------------------------------------
C|  THIS LOOP WILL EVALUATE THE NODAL INCI. FOR ELEMENT. |
C-------------------------------------------------------
       EN = 1
       LOOP = -1
       DO 20 I=1,NXDIV
          IF(I.EQ.(NXDIV1+1)) GO TO 5
          IF(I.EQ.(NXDIV1+NXDIV2+1)) GO TO 5
          LOOP = LOOP + 1
          GO TO 10
    5     LOOP = LOOP + NYNOD + 1
   10     DO 20 J = 1,NYDIV
            IND1(EN) = EN + LOOP
            IND3(EN) = IND1(EN) + NYNOD
```

```
            EN = EN + 1
   20    CONTINUE
C
C-----------------------------------------------------------
C|  THIS LOOP WILL ASSIGN THE X AND Y COORDINATES OF EACH  |
C|  NODE OF EACH ELEMENT.                                  |
C-----------------------------------------------------------
C
      COT=1./DTAN(ANG)
      IF(ANGLE.EQ.90.) COT=0.0
        NN = 0
        DO 40 I = 1,NXNOD
           DO 30 J = 1,NYNOD
              NN = NN + 1
              XCOOR(NN) = COORX(I) + COORY(J)*COT
              YCOOR(NN) = COORY(J)
   30      CONTINUE
   40    CONTINUE
C-------------------------------------------
C:  CHECK COORD. OF PARALLELOGRAM ELEMENTS. |
C-------------------------------------------
c     IF(ANGLE.EQ.90.) GO TO 1070
c     WRITE(4,1050) (XCOOR(NN),NN=1,40)
c     WRITE(4,1060) (YCOOR(NN),NN=1,40)
c 1050   FORMAT(/,' X COORD. OF PARA. ELEMENT IN NODE NO.',
c     *          /,40F12.5)
c 1060   FORMAT(/,' Y COORD. OF PARA. ELEMENT IN NODE NO.',
c     *          /,40F12.5)
c 1070   CONTINUE
 1010   FORMAT('THICKNESS ELASTIC MODULUS PR',/,
     + F9.4,6X,F10.2,5X,F4.2)
 1020 FORMAT(' X-COORDINATES')
 1030 FORMAT(' Y-COORDINATES')
 1040 FORMAT(8F10.2)
        RETURN
        END
C
C
C
C---------------------------------------------------------
C|  SUBROUTINE INPWRT WILL SAVE STRUCTURAL DISPLACEMENTS
C|  IN TABULAR FORM ON DATA FILE FOR THE INPLANE PROGRAM.
C---------------------------------------------------------
C
        SUBROUTINE INPWRT (SDELTA)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION SDELTA(NDISP),NODES(48,3)
        INTEGER BW
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
         COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                 NOE,NDISP,NNOD,BW,NBK,IRUN
C
```

```
      WRITE(7,2000) IRUN
C-----------------------------------------
C:  IF ALL THE NODES ARE TO BE PRINTED.  :
C-----------------------------------------
      WRITE(7,2005)
      READ(3,*) NUMINC,((NODES(I,J),J=1,3),I=1,NUMINC)
      IF (NODES(1,1) .EQ. 1 .AND. NODES(1,2) .EQ. NNOD
   +       NODES(1,3) .EQ. 1) THEN
        NN = 0
        DO 40 I = 1,NXNOD
          DO 30 J = 1,NYNOD
            NN = NN+1
            IW=NN*3-2
            IX=IW+1
            IY=IW+2
   WRITE(7,2010) NN,SDELTA(IW),SDELTA(IX),SDELTA(IY)
  30      CONTINUE
          WRITE(7,2020)
  40    CONTINUE
      ELSE
        DO 60 I = 1,NUMINC
          DO 50 J = NODES(I,1),NODES(I,2),NODES(I,3)
            IW=J*3-2
            IX=IW+1
            IY=IW+2
     WRITE(7,2010) J,SDELTA(IW),SDELTA(IX),SDELTA(IY)
  50      CONTINUE
          WRITE(7,2020)
  60    CONTINUE
      END IF
 2000 FORMAT ('1',79('=')/23X,'STRL DEFS FOR CASE',I2/
   +          ' ',79('=')////
   +          '  NODE',10X,'W',14X,'TX','/59('-')//)
 2005 FORMAT ('1',10X,'W IS IN INCHES')
 2010 FORMAT (' ',I4,3(4X,D14.8))
 2020 FORMAT (' ')
      RETURN
      END
C
C-------------------------------------------------------
C|  SUBROUTINE INSERT WILL INSERT THE
C|  SYMMETRIC 12 X 12 STIFF MATRIX [SK] INTO THE MATRIX
C|  [BK] WHICH IS THE  NON-ZERO PORTION OF THE
C|  STRUCTURAL STIFFNESS MATRIX. {DUMIND} GIVES THE
C|  SUBSCIPTS OF THE SYMMETRICALLY-BANDED MATRIX [BK].
C   [BK] IS STORED  AS A ONE DIMENSIONAL ARRAY.
C-------------------------------------------------------
C
      SUBROUTINE INSERT (BK,SK,IND11,IND33)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION BK(NBK),SK(12,12)
      INTEGER BW,DUMIND(12)
```

```
      COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
 COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
    +                    NOE,NDISP,NNOD,BW,NBK,IRUN
C
       DUMIND(1) = IND11 * 3 - 2
       DO 10 I = 2,6
  10     DUMIND(I) = DUMIND(I-1) + 1
       DUMIND(7) = IND33*3-2
       DO 20 J = 8,12
  20     DUMIND(J) = DUMIND(J-1) + 1
       DO 40 I = 1,12
         II = DUMIND(I)
         DO 30 J = I,12
           JJ = DUMIND(J) - II + 1
C----------------------------------------------------
C|   CALCULATE THE POSITION OF (II,JJ) IN THE
C    ONE-DIMENSIONAL ARRAY [BK] |
C----------------------------------------------------
           IIJJ = (II-1) * BW + JJ
           BK(IIJJ) = BK(IIJJ) + SK(I,J)
  30     CONTINUE
  40   CONTINUE
       RETURN
       END
C
C
C----------------------------------------------------------
C SUBROUTINE JOINT WILL INSERT THE VERT AND TORS SPRINGS |
C AT THE JOINTS TO THE STRUCTURAL STIFFNESS MATRIX [BK1] |
C----------------------------------------------------------
C
     SUBROUTINE JOINT (BK1,CONST2,CONST3,YCOOR)
     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
     DIMENSION BK1(NBK),SPRING(2),YCOOR(NNOD)
     DIMENSION NS(2),ICONST(2)
     INTEGER BW
      COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
    +                    NOE,NDISP,NNOD,BW,NBK,IRUN
C
C-----------------------------------------------------
C THE SMALLEST NODE # OF JOINT 1 & 2 , NS(1) & NS(2)|
C-----------------------------------------------------
     NS(1)=NXDIV1 *NYNOD+1
     NS(2)=(NXDIV1+NXDIV2+1)*NYNOD+1
     DO 500 I=1,2
C---------------------------------------
C          I=1   FOR JOINT #1          |
C          I=2   FOR JOINT #2          |
C---------------------------------------
       DO 500 J=1,NYDIV
         EWID = YCOOR(NS(I)+J)-YCOOR(NS(I)+J-1)
```

```
C-----------------------------------------------------
C THE VERT SPRING CONSTANT,SPRING(1) & THE TORSIONAL|
C SPRING CONSTANT, SPRING(2)                         |
C-----------------------------------------------------
          SPRING(1)=CONST2*EWID/2.
          SPRING(2)=CONST3*EWID/2.
C-----------------------------------------------------
C   ICONST(1) IS TO BE USED  VERTICAL SPRING (K=1) & |
C   ICONST(2) FOR TORSIONAL SPRING (K=2)             |
C-----------------------------------------------------
          ICONST(1)=0
          ICONST(2)=2
          DO 500 K=1,2
C-------------------------------------------------------
C          L=1    FOR UPPER NODE OF ELEMENT            |
C          L=2    FOR LOWER NODE OF ELEMENT            |
C-------------------------------------------------------
           DO 500 L=1,2
C-------------------------------------------------------
C CALCULATE THE STRUCTURAL COORD. OF LEFT NODE, IL, & |
C THAT OF THE RIGHT NODE, IR                           |
C-------------------------------------------------------
             IL=(NS(I)+J+L-3)*3+1+ICONST(K)
             IR=IL + (NYNOD*3)
C---------------------------------------------------------
C ADD THE SPRING CONSTANTS TO THE APP. POSITIONS IN THE |
C STRUCTURE STIFFNESS MATRIX [BK1]                       |
C---------------------------------------------------------
             ILBK=(IL -1)*BW+1
             IRBK=(IR-1)*BW+1
             BK1(ILBK)=BK1(ILBK)+SPRING(K)
             BK1(IRBK)=BK1(IRBK)+SPRING(K)
C-----------------------------------------------------
C    ADD THE NEGATIVE OF THE SPRING CONSTANTS TO THE
C    APPROPRIATE POSITIONS IN [BK1]
C-----------------------------------------------------
             JRBK=ILBK+NYNOD*3
             BK1(JRBK)=BK1(JRBK)-SPRING(K)
  500 CONTINUE
      RETURN
      END
C
C
C--------------------------------------------------------
C SUBROUTINE KMAT WILL INITIALIZE THE ELEM. STIFF MATRIX
C USING THE [SK1], [SK2], [SK3], AND [SK4] THAT ARE
C INITIALIZED BY INTERNAL DATA STATEMENTS. [SK1] AND [SK2]
C HAVE BEEN ADJUSTED FOR THE WIDTH AND LENG.OF THE ELEM.
C--------------------------------------------------------
C
        SUBROUTINE KMAT (SK,ELEN,EWID)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
```

```
      DIMENSION SK(12,12)
        INTEGER BW
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
  COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
     +                  NOE,NDISP,NNOD,BW,NBK,IRUN
C
        DO 10 I=1,11
      DO 10 J=I+1,12
   10    SK(I,J) = 0.0
C-----------------------------------------------------
C   DEFINE THE CONSTANTS USED FOR STIFFNESS MATRIX
C-----------------------------------------------------
C
      A = ELEN * 0.5
      B = EWID * 0.5
C
      A1 = 1./ A
      A2 = 1./(A * A)
      A3 = 1./(A * A * A)
      A4 = 1./(A * A * A * A)
        B1 = 1./ B
      B2 = 1./(B * B)
      B3 = 1./(B * B * B)
      B4 = 1./(B * B * B * B)
C
      COT = 1./ TAN(ANG)
      CSC = 1./ SIN(ANG)
      IF(ANGLE.EQ.90.) COT=0.0
C
      COT2 = COT**2
        COT3 = COT**3
      COT4 = COT**4
      CSC2 = CSC**2
      CSC3 = CSC**3
      CSC4 = CSC**4
C
C---( for SK2 )
C
      C1C1 = A4 * COT4
      C2C2 = B4
      C3C3 = A2 * B2 * COT2
C
      C6C6 = A2 * COT2 * CSC2
      C7C7 = B2 * CSC2
      C6C7 = A1 * B1 * COT * CSC2
C
      C1C7 = A2 * B1 * COT2 * CSC
      C2C6 = A1 * B2 * COT *CSC
      C2C7 = B3 * CSC
C
      C6C9 = A2 * COT3 * CSC
      C7C9 = A1 * B1 * COT2 * CSC
```

```
      C7C10= B2 * COT * CSC
      C9C9 = A2 * COT4
      C10C10 = B2 * COT2
      C9C10 = A1 * B1 * COT3
C
      C1C9 = A3 * COT4
      C1C10 = A2 * B1 * COT3
      C2C9 = A1 * B2 * COT2
      C2C10 = B3 * COT
C
C---( for SK3 )
C
      C1 = A4 * COT2
      C2 = A2 * B2
      C7 = A2 * B1 * CSC
C
      C1D = A3 * COT2
      C2D = A1 * B2
      C3D = A2 * B1 * COT
      DC6 = A2 * COT * CSC
      DC7 = A1 * B1 * CSC
      Z4 = A2 * COT2
      Z5 = A1 * B1 * COT
C
C---( for SK4 )
C
      H44 = A4 * COT2
      H55 = A2 * B2
      H45 = A3 * B1 * COT
      H48 = A3 * COT * CSC
      H58 = A2 * B1 * CSC
C
      P88 = A2 * CSC2
      P81 = A2 * COT * CSC
      P82 = A1 * B1 * CSC
      Q11 = A2 * COT2
      Q22 = B2
      Q12 = A1 * B1 * COT
C
C-------------------------------------------------------
C
C       ELEMENT  STIFFNESS MATRIX
C
CSK = ab.sin(a) [ SK1.Dx + SK2.Dy + SK3.D1 + SK4.Dxy ]
C
C          for isotropic material,    Dx = Dy = 1
C                                    D1 = PR(poisson's ratio)
C                                    Dxy = (1. - PR ) /2.
C
C              multiplication factor, S =  ab.sin(a) * D
C
C                                      where a= ELEN/2.
```

```
C                                   b= EWID/(sin(a)*2.)
C                                   D= E*T**3/12(1-PR**2)
C
C                                   ELEN*EWID*E*T**3
C                               S = ---------------
C                                   48.* (1.- PR**2)
C
C-----------------------------------------------------
C
C
      DXY = (1.-PR)/2.
C
SK(1,1) = A4 + (C1C1+C2C2+1.9*C3C3) + PR*(2*C1+0.5*C2)
     *                + DXY*(4*H44+(7./5.)*H55)
SK(2,1) = (-0.7*C1C7-C2C7) + PR*(-0.5*C7)+DXY*(-0.2*H58)
SK(2,2) = 0. + ((8./15.)*C6C6+(4./3.)*C7C7-C6C7) + 0.
     *                + DXY*(8./15.)*P88
SK(3,1) = A3 + (C1C9+0.7*C2C9) + PR*(2.*C1D+0.5*C2D)
     *                + DXY*(4.*C1D+0.2*C2D)
SK(3,2) = ((1./6.)*C6C9-C7C9+(1./6.)*C7C10)+ PR*((1./6.)
     *             *DC6-DC7) + DXY*(1./3.)*P81
SK(3,3) = (4./3.)*A2 + ((4./3.)*C9C9+(8./15.)*C10C10-(1.)*
     *     C9C10) + PR*((8./3.)*Z4-Z5) + DXY*((16./3.)*Q11
     *                + (8./15.)*Q22-2.*Q12)
SK(4,1) = A4 + (0.5*C1C1-C2C2-1.9*C3C3) + PR*(C1-0.5*C2)
     *                + DXY*(2.*H44-(7./5.)*H55)
SK(4,2) = 0. + (0.2*C1C7+C2C7) + 0. + DXY*0.2*H58
SK(4,3) =  (0.5*C1C9-C1C10-0.7*C2C9) + PR*(C1D-C3D-0.5
     *              *C2D) + DXY*(2.*C1D-2.*C3D-0.2*C2D)
SK(4,4) = A4 + (C1C1+C2C2+1.9*C3C3) + PR*(2.*C1+0.5*C2)
     *                + DXY*(4*H44+(7./5.)*H55)
SK(5,1) = 0. + (-0.2*C1C7-C2C7) + 0. + DXY*(-0.2*H58)
SK(5,2) = 0. + ((-2./15.)*C6C6+(2./3.)*C7C7) + 0.
     *                + DXY*(-2./15.)*P88
SK(5,3) = (-1./6.)*C6C9-(1./6.)*C7C10 + PR*(-1./6.)*DC6
     *                + DXY*(-1./3.)*P81
SK(5,4) = 0. + (0.7*C1C7+C2C7) + PR*0.5*C7 + DXY*0.2*H58
SK(5,5) = 0. + ((8./15)*C6C6+(4./3.)*C7C7+C6C7) + 0.
     *               + DXY*(8./15.)*P88
SK(6,1) = (0.5*C1C9+C1C10-0.7*C2C9) + PR*(C1D-0.5*C2D
     *             + C3D) + DXY*(2.*C1D+2.*C3D-0.2*C2D)
SK(6,2) = (-1./6.)*C6C9-(1./6.)*C7C10 + PR*(-1./6.)*DC6
     *                + DXY*(-1./3.)*P81
SK(6,3) = A2 + ((2./3.)*C9C9-(8./15)*C10C10) + PR*
     *        (4./3.)*Z4 + DXY*((8./3.)*Q11-(8./15.)*Q22)
SK(6,4) = A3 + (C1C9+0.7*C2C9) + PR*(2.*C1D+0.5*C2D)
     *            + DXY*(4.*C1D+0.2*C2D)
SK(6,5) = 0. + ((1./6.)*C6C9+C7C9+(1./6.)*C7C10)
     *           + PR*((1./6.)*DC6+DC7) + DXY*(1./3.)*P81
SK(6,6) = A2 + ((4./3.)*C9C9+(8./15.)*C10C10+C9C10)
     *     ((8./3.)*Z4+Z5) + DXY*((16./3.)*Q11+(8./15.)
     *                  *Q22+2.*Q12)
```

```
SK(7,1) = (-C1C1+0.5*C2C2-1.9*C3C3) + PR*(-2.*C1-0.5*C2)
     *                 + DXY*(-4.*H44-(7./5.)*H55)
SK(7,2) = 0. + (0.7*C1C7+C2C6-0.5*C2C7) + PR*(0.5)*C7
     *                 + DXY*(0.2*H58)
SK(7,3) = -A3 + (-C1C9-0.2*C2C9) + PR*(-2.*C1D)
     *                 + DXY*(-4.*C1D-0.2*C2D)
SK(7,4) = A4 + (-0.5*C1C1-0.5*C2C2+1.9*C3C3) + PR*(-C1
     *            + 0.5*C2) + DXY*(-2.*H44+(7./5.)*H55)
SK(7,5) = (0.2*C1C7-C2C6-0.5*C2C7) + 0. + DXY*0.2*H58
SK(7,6) =  (-0.5*C1C9-C1C10+0.2*C2C9) + PR*(-C1D-C3D)
     *                 + DXY*(-2.*C1D-2.*C3D+0.2*C2D)
SK(7,7) =  (C1C1+C2C2+1.9*C3C3) + PR*(2.*C1+0.5*C2)
     *                 + DXY*(4.*H44+(7./5.)*H55)
SK(8,1) = (0.7*C1C7-C2C6-0.5*C2C7) + PR*0.5*C7 + DXY*0.2
     *                  *H58
SK(8,2) = 0. + ((-8./15.)*C6C6+(2./3.)*C7C7) + 0.
     *                 + DXY*(-8./15.)*P88
SK(8,3) =  (-1./6.)*C6C9-(1./6.)*C7C10 + PR*(-1./6.)
     *                 *DC6 + DXY*(-1./3.)*P81
SK(8,4) = C1C7+C2C6+0.5*C2C7) + 0. + DXY*(-0.2)*H58
SK(8,5) =  ((2./15.)*C6C6+(1./3.)*C7C7+C6C7) + 0.
     *                 + DXY*(2./15.)*P88
SK(8,6) =  (1./6.)*C6C9+(1./6.)*C7C10 + PR*(1./6.)
     *                  *DC6 + DXY*(1./3.)*P81
SK(8,7) = 0. + (-0.7*C1C7-C2C7) + PR*(-0.5*C7)
     *                 + DXY*(-0.2*H58)
SK(8,8) =  ((8./15.)*C6C6+(4./3.)*C7C7+C6C7) + 0.
     *                 + DXY*(8./15.)*P88
SK(9,1) = C9+0.2*C2C9) + PR*2.*C1D + DXY*(4.*C1D+0.2
     *                  *C2D)
SK(9,2) = 0. + (-1./6.)*C6C9+(-1./6.)*C7C10
     *           + PR*(-1./6.)*DC6 + DXY*(-1./3.)*P81
SK(9,3) = A2 + ((2./3.)*C9C9-(2./15.)*C10C10) + PR*
     *      (4./3.)*Z4 + DXY*((8./3.)*Q11-(2./15.)*Q22)
SK(9,4) =  (0.5*C1C9+C1C10-0.2*C2C9) + PR*(C1D+C3D)
     *                 + DXY*(2*C1D+2*C3D-0.2*C2D)
SK(9,5) = 0. + (1./6.)*C6C9+(1./6.)*C7C10
*                + PR*(1./6.)*DC6 + DXY*(1./3.)*P81
SK(9,6) = A2 + ((1./3.)*C9C9+(2./15.)*C10C10+C9C10)
     *      Z4+Z5) + DXY*((4./3.)*Q11+(2./15.)
     *                  *Q22+2.*Q12)
SK(9,7) = -A3 + (-C1C9-0.7*C2C9) + PR*(-2.*C1D-0.5*C2D)
     *                 + DXY*(-4.*C1D-0.2*C2D)
SK(9,8) = 0. + ((1./6.)*C6C9+C7C9+(1./6.)*C7C10)
     *       + PR*((1./6.)*DC6+DC7) + DXY*(1./3.)*P81
SK(9,9) = A2 + ((4./3.)*C9C9+(8./15.)*C10C10+C9C10)
     * PR*((8./3.)*Z4+Z5) + DXY*((16./3.)*Q11+(8./15.)
     *                  *Q22+2.*Q12)
SK(10,1) = (-0.5*C1C1-0.5*C2C2+1.9*C3C3) + PR*(-C1
     *        + 0.5*C2) + DXY*(-2.*H44+(7./5.)*H55)
SK(10,2) =C1C7-C2C6+0.5*C2C7) + 0. + DXY*(-0.2*H58)
SK(10,3) = (-0.5*C1C9+C1C10+0.2*C2C9) + PR*(-C1D
```

```
     *        +C3D) + DXY*(-2.*C1D+2.*C3D+0.2*C2D)
SK(10,4) =0.5*C2C2-1.9*C3C3) + PR*(-2.*C1-0.5*C2)
     *                  + DXY*(-4.*H44-(7./5.)*H55)
SK(10,5) =(-0.7*C1C7+C2C6+0.5*C2C7) + PR*(-0.5*C7)
     *                  + DXY*(-0.2*H58)
SK(10,6) = (-C1C9-0.2*C2C9) + PR*(-2.*C1D) + DXY*
     *                  (-4.*C1D-0.2*C2D)
SK(10,7) =(0.5*C1C1-C2C2-1.9*C3C3) + PR*(C1-0.5*C2)
     *                  + DXY*(2.*H44-(7./5.)*H55)
SK(10,8) = (0.2*C1C7+C2C7) + 0. + DXY*0.2*H58
SK(10,9) = (-0.5*C1C9-C1C10+0.7*C2C9) + PR*(-C1D-C3D+
     *     0.5*C2D) + DXY*(-2.*C1D-2.*C3D+0.2*C2D)
SK(10,10) = (C1C1+C2C2+1.9*C3C3) + PR*(2.*C1+0.5*C2)
     *                  + DXY*(4.*H44+(7./5.)*H55)
SK(11,1) =C1C7+C2C6-0.5*C2C7) + 0. + DXY*(0.2*H58)
SK(11,2) =  ((2./15.)*C6C6+(1./3.)*C7C7-C6C7) + 0.
     *                  + DXY*(2./15.)*P88
SK(11,3) = 0. + (1./6.)*C6C9+(1./6.)*C7C10
     *            + PR*(1./6.)*DC6 + DXY*(1./3.)*P81
SK(11,4) =  (-0.7*C1C7-C2C6+0.5*C2C7) + PR*(-0.5*C7)
     *                 + DXY*(-0.2*H58)
SK(11,5) =  ((-8./15.)*C6C6+(2./3.)*C7C7) + 0. + DXY*
     *                    (-8./15.)*P88
SK(11,6) = 0. + (-1./6.)*C6C9-(1./6.)*C7C10
     *            + PR*(-1./6.)*DC6 + DXY*(-1./3.)*P81
SK(11,7) = 0. + (-0.2*C1C7-C2C7) + 0. + DXY*(-0.2*H58)
SK(11,8) =  ((-2./15.)*C6C6+(2./3.)*C7C7) + 0. + DXY*
     *                  (-2./15)*P88
SK(11,9) = C6C9-(1./6.)*C7C10+ PR*(-1./6.)*DC6
     *                  + DXY*(-1./3.)*P81
SK(11,10) =C1C7+C2C7) + PR*(0.5*C7) + DXY*(0.2*H58)
SK(11,11) = ((8./15.)*C6C6+(4./3.)*C7C7-C6C7) + 0.
     *                  + DXY*(8./15.)*P88
SK(12,1) =  (0.5*C1C9-C1C10-0.2*C2C9) + PR*(C1D-C3D)
     *                 + DXY*(2.*C1D-2.*C3D-0.2*C2D)
SK(12,2) = 0. + (1./6.)*C6C9+(1./6.)*C7C10
     *             + PR*(1./6.)*DC6 + DXY*(1./3.)*P81
SK(12,3) = A2 + ((1./3.)*C9C9+(2./15.)*C10C10-C9C10)
     *      2./3.)*Z4-Z5) + DXY*((4./3.)*Q11+(2./15.)
     *                    *Q22-2.*Q12)
SK(12,4) = (C1C9+0.2*C2C9) + PR*(2.*C1D) + DXY*(4.*C1D
     *                  + 0.2*C2D)
SK(12,5) = 0. + (-1./6.)*C6C9-(1./6.)*C7C10
     *           + PR*(-1./6.)*DC6 + DXY*(-1./3.)*P81
SK(12,6) =A2 + ((2./3.)*C9C9-(2./15.)*C10C10) + PR*
     *     (4./3.)*Z4 + DXY*((8./3.)*Q11-(2./15.)*Q22)
SK(12,7) = (-0.5*C1C9+C1C10+0.7*C2C9) + PR*(-C1D
     *     *C2D+C3D) + DXY*(-2.*C1D+2.*C3D+0.2*C2D)
SK(12,8) = 0. + (-1./6.)*C6C9-(1./6.)*C7C10
     *      + PR*(-1./6.)*DC6 + DXY*(-1./3.)*P81
SK(12,9) =A2 + ((2./3.)*C9C9-(8./15)*C10C10) + PR*
     *     Z4 + DXY*((8./3.)*Q11-(8./15.)*Q22)
```

```
SK(12,10)=(-C1C9-0.7*C2C9) + PR*(-2.*C1D-0.5*C2D)
     *                   + DXY*(-4.*C1D-0.2*C2D)
SK(12,11)= 0. + ((1./6.)*C6C9-C7C9+(1./6.)*C7C10)
     *                      + PR*((1./6.)*DC6-DC7)
     *                  + DXY*(1./3.)*P81
SK(12,12)= SK(3,3)
C
C
      DO 100 I=1,12
      DO 100 J=1,I
         IF(I.EQ.J) GO TO 100
         SK(J,I) = SK(I,J)
  100   CONTINUE
C
      D = E*T*T*T/(12.*(1.-PR*PR))
      S = D*ELEN*EWID/4.
C
      CALL SMULT(SK,S,SK,12,12)
        RETURN
```

```
C---------------------------------------------------
C   SUBROUTINE LOCALF IS TO FIND LOCAL STRESSES
C   AND MOMENTS ALONG THE FACE OF THE SKEWED
C   JOINT USING THE MOHR'S CIRCLE.
C---------------------------------------------------
C
      SUBROUTINE LOCALF(GLOBAL,LOCAL)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION LOCAL(NDISP)
        DIMENSION GLOBAL(NDISP)
        INTEGER BW
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
         COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                  NOE,NDISP,NNOD,BW,NBK,IRUN
C
      PI = 4.0*ATAN(1.0)
      A2= (PI/2.-ANG)*2
      IF(ANGLE.EQ.90.) A2=0.0
C
      DO 10 I=1,NNOD
         I1 = I*3-2
         I2 = I1+1
         I3 = I1+2
C
         GX = GLOBAL(I1)
         GY = GLOBAL(I2)
           GXY= GLOBAL(I3)
C
      R = SQRT(0.25*(GX-GY)**2+GXY**2)
      RR=(GX-GY)/(2.*R)
C
      B2= ACOS(RR)
      TRAD=B2+A2
C
      LOCAL(I1) = 0.5*(GX+GY)+R*COS(TRAD)
      LOCAL(I2) = 0.5*(GX+GY)-R*COS(TRAD)
      LOCAL(I3) = R*SIN(TRAD)
   10   CONTINUE
      RETURN
      END
C
C
C-------------------------------------------------------------
C SUBROUTINE MAXMUM WILL FIND THE MAX. OF THE MATRIX [X].
C THE MAXIMUM VALUE WILL BE FOUND BETWEEN  INCREMENTS.
C-------------------------------------------------------------
C
        SUBROUTINE MAXMUM (X)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION X(NDISP)
        DOUBLE PRECISION MAX
        INTEGER NODE(44,3),BW
```

```
      CHARACTER*80 COMAND,TITLE,DIR*7,VALTIT*14
     COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
       COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                NOE,NDISP,NNOD,BW,NBK,IRUN
       COMMON /CHAR/ COMAND
C
      WRITE (4,2000)
      IF (COMAND(15:25) .EQ. 'DEFLECTIONS') THEN
          TITLE = '     MAXIMUM NODAL DEFLECTIONS FOR '
          VALTIT = ' DEFLECTIONS  '
        ELSE IF (COMAND(15:21) .EQ. 'MOMENTS') THEN
          TITLE = '         MAXIMUM NODAL MOMENTS FOR '
          VALTIT = '   MOMENTS    '
        ELSE IF (COMAND(15:22) .EQ. 'STRESSES') THEN
          TITLE = '        MAXIMUM NODAL STRESSES FOR '
          VALTIT = '   STRESSES   '
        ELSE
          TITLE = '    MAXIMUM PRINCIPAL STRESSES FOR '
          VALTIT = 'PRIN. STRESSES'
      END IF
      READ (3,*) NUMINC,((NODE(I,J),J=1,3),I=1,NUMINC)
      DO 30 I = 1,NUMINC
        WRITE (4,2010) TITLE,NODE(I,1),NODE(I,2),IRUN,
        DO 20 K = 1,3
          MAX = 0.0
          DO 10 J = NODE(I,1),NODE(I,2),NODE(I,3)
            IWXY=(J-1)*3+K
            IF (ABS(MAX) .LT. ABS(X(IWXY))) THEN
              MAX = X(IWXY)
              MAXNOD = J
            END IF
10        CONTINUE
          IF (COMAND(15:21) .EQ. 'DEFLECT') THEN
            DIR = '   W   '
            IF (K .EQ. 2) DIR = 'THETA X'
            IF (K .EQ. 3) DIR = 'THETA Y'
          ELSE IF (COMAND(15:21) .EQ. 'MOMENTS') THEN
            DIR = '  MXX  '
            IF (K .EQ. 2) DIR = '  MYY  '
            IF (K .EQ. 3) DIR = '  MXY  '
          ELSE IF (COMAND(15:21) .EQ. 'PRINCIP') THEN
            DIR = '  S1  '
            IF (K .EQ. 2) DIR = '  S2   '
            IF (K .EQ. 3) DIR = ' TMAX  '
          ELSE
            DIR = '  SXX '
            IF (K .EQ. 2) DIR = '  SYY  '
            IF (K .EQ. 3) DIR = '  TXY  '
          END IF
          WRITE (4,2020) MAXNOD,DIR,MAX
20      CONTINUE
        WRITE (4,2030)
```

```
   30      CONTINUE
 2000      FORMAT ('1')
 2010      FORMAT (' ',79('=')/' ',A40,I4,' TO ',I4,',I3/
     +       ' ',79('=')//' ','NODE   DIRECTION     '/)
 2020      FORMAT (' ',I4,4X,A7,D18.8)
 2030      FORMAT (///)
           RETURN
           END
C
C
C---------------------------------------------------
C SUBROUTINE MOMWRT WILL PRINT OUT THE AVERAGE OR |
C STRESS-INTENSITIES FOR EACH NODE OF THE STRUCTURE.
C---------------------------------------------------
C
           SUBROUTINE MOMWRT (FMON)
           IMPLICIT DOUBLE PRECISION (A-H,O-Z)
           DIMENSION FMON(NDISP)
           INTEGER NODES(48,3),BW
           CHARACTER COMAND*80,HEAD*39,TITLE*17
          COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
            COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +        NOE,NDISP,NNOD,BW,NBK,IRUN
            COMMON /CHAR/ COMAND
C
           TITLE = 'MOMENTS FOR CASE'
           HEAD = 'MXX               MYY                 MXY'
           IF (COMAND(7:14) .EQ. 'STRESSES') THEN
             HEAD = 'SXX               SYY                 TXY'
             TITLE = 'STRESSES FOR CASE'
           END IF
c----------------------------------------------------------
         IF(COMAND(13:20).EQ.'STRESSES') THEN
             HEAD = 'SXX               SYY                 TXY'
             TITLE = 'STRESSES FOR CASE'
           END IF
c----------------------------------------------------------
           READ(3,*) NUMINC,((NODES(I,J),J=1,3),I=1,NUMINC)
           WRITE(4,*)NUMINC,((NODES(I,J),J=1,3),I=1,NUMINC)
           WRITE(4,2000) TITLE,IRUN,HEAD
           IF (NODES(1,1) .EQ. 1 .AND. NODES(1,2) .EQ. NNOD .
     +        NODES(1,3) .EQ. 1) THEN
             NN=0
             DO 40 K = 1,NXNOD
               DO 30 I = 1,NYNOD
                 NN = NN+1
                 IW=NN*3-2
                 IX=IW+1
                 IY=IW+2
                 WRITE(4,2010) NN,FMON(IW),FMON(IX),FMON(IY)
   30          CONTINUE
               WRITE(4,2020)
```

```
   40       CONTINUE
          ELSE
            DO 60 I = 1,NUMINC
              DO 50 J = NODES(I,1),NODES(I,2),NODES(I,3)
                IW=J*3-2
                IX=IW+1
                IY=IW+2
                WRITE(4,2010) J,FMON(IW),FMON(IX),FMON(IY)
   50         CONTINUE
              WRITE(4,2020)
   60       CONTINUE
          END IF
 2000     FORMAT ('1',79('=')/' ',25X,'AVERAGE ',A17,I2/
     +             ' ',79('=')
     +            ///' ','NODE #',7X,
     +            13X,A39/' ',79('-')//)
 2010     FORMAT (' ',I4,13X,3D18.8)
 2020     FORMAT (' ')
          RETURN
          END
C
C
C-------------------------------------------------------------
C SUBROUTINE MULT MULTIPLY  N1 X N2 MATRIX [A] TIMES THE     |
C N2 X N3 MATRIX [B] AND YIELD THE N1 X N3 MATRIX [C].       |
C-------------------------------------------------------------
C
          SUBROUTINE MULT(A,B,C,N1,N2,N3)
          IMPLICIT DOUBLE PRECISION (A-H,O-Z)
          DIMENSION A(N1,N2),B(N2,N3),C(N1,N3)
          DO 30 I = 1,N1
             DO 20 J = 1,N3
                C(I,J) = 0.0
                DO 10 K =1,N2
                   C(I,J) = C(I,J) + A(I,K)*B(K,J)
   10           CONTINUE
   20        CONTINUE
   30     CONTINUE
          RETURN
          END
C
C
C--------------------------------------------------------------
C SUBROUTINE PRSTRS WILL COMPUTE PRINCIPAL STRESSES S1,S2,
C AND THETA; THE ANGLE OF THE PRINCIPAL STRESS PLANE. THESE
C CALCULATIONS ARE BASED ON A MOHR CIRCLE REPRESENTATION OF
C CONFIGURATION.
C     R  = RADIUS OF THE MOHR CIRCLE.
C     OC = CENTER OF THE MOHR CIRCLE ON THE X-AXIS.
C--------------------------------------------------------------
C
          SUBROUTINE PRSTRS (NODSTR,PRNSTR,THETA)
```

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION NODSTR(NDISP)
      DIMENSION PRNSTR(NDISP),THETA(NNOD)
    INTEGER BW
     COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
       COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                NOE,NDISP,NNOD,BW,NBK,IRUN
      DO 10 I = 1,NNOD
        I1=I*3-2
        I2=I1+1
        I3=I1+2
        SXX = NODSTR(I1)
        SYY = NODSTR(I2)
        TXY = NODSTR(I3)
        R = SQRT(((SXX-SYY)/2)**2 + TXY**2)
        OC = (SXX+SYY)/2
        PRNSTR(I1) = OC+R
        PRNSTR(I2) = OC-R
        PRNSTR(I3) = R
        THETA(I) = ATAN(2*TXY/(SXX-SYY))/2.0
 10   CONTINUE
      RETURN
      END
C
C
C---------------------------------------------------------
C SUBROUTINE PSWRT WILL PRINT  PRINCIPAL STRESSES FOR ANY|
C GIVEN INCREMENT OF NODES. THERE IS AN OPTION TO PRINT  |
C ANGLE THETA IN DEGREES OR RADIANS.                     |
C---------------------------------------------------------
C
      SUBROUTINE PSWRT (PRNSTR,THETA)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION PRNSTR(NDISP),THETA(NNOD)
      INTEGER NODES(48,3),BW,DEG
      CHARACTER DEGHD*5
     COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
       COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,
     +                NOE,NDISP,NNOD,BW,NBK,IRUN
C
   READ(3,*) NUMINC,DEG,((NODES(I,J),J=1,3),I=1,NUMINC)
      IF (DEG .EQ. 1) THEN
C-------------------------------------------------
C|  CONVERT VECTOR {THETA} FROM RADIANS TO DEGREES. |
C-------------------------------------------------
        DEGHD = '(DEG)'
        PI = 180/3.141592654
        DO 5 I = 1,NNOD
          THETA(I) = THETA(I)*PI
  5     CONTINUE
      ELSE
        DEGHD = '(RAD)'
```

```
        END IF
        WRITE(4,2000) IRUN,DEGHD
  IF (NODES(1,1) .EQ. 1 .AND. NODES(1,2) .EQ. NNOD .AND.
     +    NODES(1,3) .EQ. 1) THEN
          NN = 0
          DO 20 K = 1,NXNOD
            DO 10 I = 1,NYNOD
              NN = NN+1
              I1=NN*3-2
              I2=I1+1
              I3=I1+2
WRITE(4,2010) NN,PRNSTR(I1),PRNSTR(I2),PRNSTR(I3),THETA(NN)
  10        CONTINUE
            WRITE(4,2020)
  20      CONTINUE
        ELSE
          DO 40 I = 1,NUMINC
            DO 30 J = NODES(I,1),NODES(I,2),NODES(I,3)
              I1=J*3-2
              I2=I1+1
              I3=I1+2
WRITE(4,2010) J,PRNSTR(I1),PRNSTR(I2),PRNSTR(I3),THETA(J)
  30        CONTINUE
            WRITE(4,2020)
  40      CONTINUE
        END IF
1000    FORMAT (A80)
2000  FORMAT ('1',79('=')/'PRINCIPAL STRESSES FOR LOAD
     + CASE',I3/' ',79('=')///' ',
     +      'NODE # ',11X,'S1          S2             TMAX
     +        THETA',A5/' ',79('-')//)
2010    FORMAT (' ',I4,4X,3D17.7,3X,D17.6)
2020    FORMAT (' ')
        RETURN
        END
C
C
C---------------------------------------------
C SUBROUTINE SLABWT CALCULATES BF1 CAUSED BY |
C THE WEIGHT OF THE SLAB                     |
C---------------------------------------------
C
      SUBROUTINE SLABWT(BF1,IND1,IND3,XCOOR,YCOOR,DENSTY)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    DIMENSION IND1(NOE),IND3(NOE),XCOOR(NNOD),YCOOR(NNOD)
      DIMENSION BF1(NDISP)
      INTEGER BW,EN
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
    COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
     +                  NOE,NDISP,NNOD,BW,NBK,IRUN
C
      DO 10 EN=1,NOE
```

```
C-----------------------------------------------------
C CALCULATE ELEMENT'S LENGTH, WIDTH AND INCIDENCES   |
C-----------------------------------------------------
        ELEN=XCOOR(IND3(EN))-XCOOR(IND1(EN))
        EWID=YCOOR(IND1(EN)+1)-YCOOR(IND1(EN))
        II=IND1(EN)*3-2
        JJ=II+3
        KK=IND3(EN)*3-2
        LL=KK+3
C---------------------------------------------------------
C CALCULATE NODAL FORCES INDUCED BY THE UNIFORM LOAD OF|
C THE WEIGHT OF THE SLAB                                  |
C---------------------------------------------------------
        W=DENSTY*T/1728000.
        TLOAD=W*ELEN*EWID
            F1 = TLOAD/4.0
            F2 = TLOAD*EWID/(24.0*SIN(ANG))
            F3 = TLOAD*ELEN/24.0
C----------------------------------------------------------
C|  INSERT THE FORCE OR COUPLE INTO THE PROPER POSITION |
C|  IN {BF1}.                                            |
C----------------------------------------------------------
            BF1(II)   = BF1(II)+F1
            BF1(II+1) = BF1(II+1)-F2
            BF1(II+2) = BF1(II+2)+F3
            BF1(JJ)   = BF1(JJ)+F1
            BF1(JJ+1) = BF1(JJ+1)+F2
            BF1(JJ+2) = BF1(JJ+2)+F3
            BF1(KK)   = BF1(KK)+F1
            BF1(KK+1) = BF1(KK+1)-F2
            BF1(KK+2) = BF1(KK+2)-F3
            BF1(LL)   = BF1(LL)+F1
            BF1(LL+1) = BF1(LL+1)+F2
            BF1(LL+2) = BF1(LL+2)-F3
   10    CONTINUE
         RETURN
         END
C
C
C---------------------------------------------------------
C  SUBROUTINE SMULT WILL MULTIPLY THE M X N MATRIX [A]
C  BY THE SCALAR S  YEILDING THE RESULTANT MATRIX [B].
C---------------------------------------------------------
C
        SUBROUTINE SMULT(A,S,B,M,N)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION A(M,N),B(M,N)
        DO 20 I = 1,M
           DO 10 J = 1,N
              B(I,J) = A(I,J)*S
   10      CONTINUE
   20   CONTINUE
```

```
      RETURN
      END
C
C
C-----------------------------------------------------------
C SUBROUTINE STRESS CALCULATE AVERAGE STRESSES FOR EACH
C OF THE STRUCTURE. CALCULATIONS ARE BASED ON MC/I
C EQUATIONS  STRESS.
C-----------------------------------------------------------
C
      SUBROUTINE STRESS (NODMOM,NODSTR)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION NODMOM(NDISP),NODSTR(NDISP),NS
      INTEGER BW
      CHARACTER COMAND*80
     COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
 COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
  +                 NOE,NDISP,NNOD,BW,NBK,IRUN
       COMMON /CHAR/ COMAND
C
      PS = 6./T**2
      NS = -1.*PS
      DO 20 I = 1,NNOD
          I1=I*3-2
          I2=I1+1
          I3=I1+2
          NODSTR(I1)=NODMOM(I1)*PS
          NODSTR(I2)=NODMOM(I2)*PS
          NODSTR(I3)=NODMOM(I3)*NS
 20   CONTINUE
      RETURN
      END
C
C
C-----------------------------------------------------------
C|  SUBROUTINE STRMAT INITIALIZE 12 X 12 STRESS MATRIX
C|  WHICH IS COMBINATION OF TRANSFORMATION MATRIX [B] AND
C|  E. MATRIX [D] FOR EACH OF ELEMENT. [SMAT] IS DEPENDE
C|  THE ELEMENT'S LENGTH, WIDTH, AND POISSON`S RATIO.
C|
C|  ELEN  = LENGTH OF ELEMENT.
C|  EWID  = WIDTH OF ELEMENT.
C|  A     = ELEN / 2
C|  B     = EWID / (2.*sin(a))
C|  PR    = POISSON'S RATIO
C|  DXY   = (1 - PR)/2
C-----------------------------------------------------------
C
     SUBROUTINE STRMAT(SMAT,ELEN,EWID,E,T,PR,ANGLE,ANG)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
     DIMENSION SMAT(12,12)
C
```

```
C--------------------------
C    DEFINE THE CONSTANTS
C--------------------------
C
      A = ELEN/2.
      B = EWID/2.
      A1= 1./A
      A2= A1 * A1
      B1= SIN(ANG)/B
      B2= B1 * B1
      D = E*T*T*T/(12.*(1.-PR*PR))
      DXY=(1.-PR)/2.
C
      COT=1./TAN(ANG)
      CSC=1./SIN(ANG)
      IF(ANGLE.EQ.90.) COT=0.0
      COT2=COT*COT
      CSC2=CSC*CSC
C
      C1  = A2*COT2
      C2  = B2*CSC2
      C3  = A1*B1*COT*CSC
      C4  = A2*COT
      C5  = A1*B1*CSC
      C6  = A1*COT*CSC
      C7  = B1*CSC2
      C8  = A1*CSC
      C9  = A1*COT2
      C10= B1*COT*CSC
      C11= A1*COT
      C12= B1*CSC
C
      CALL ZERO(SMAT,12,12)
C
      SMAT(1,1)  = 1.5*A2 + PR*(1.5*C1+1.5*C2-0.5*C3)
      SMAT(1,2)  = PR*(C6-2.*C7)
      SMAT(1,3)  = 2.*A1 + PR*(2.*C9-C10)
      SMAT(1,4)  = PR*(-1.5*C2+0.5*C3)
      SMAT(1,5)  = PR*(-C7)
      SMAT(1,6)  = PR*(2.*C9+C10)
      SMAT(1,7)  = -1.5*A2 + PR*(-1.5*C1+0.5*C3)
      SMAT(1,8)  = PR*(-C6)
      SMAT(1,9)  = A1 + PR*C9
      SMAT(1,10)= PR*(-0.5*C3)
C
      SMAT(2,1)  = PR*(1.5*A2) + 1.5*C1+1.5*C2-0.5*C3
      SMAT(2,2)  = C6-2.*C7
      SMAT(2,3)  = PR*2.*A1 + 2.*C9-C10
      SMAT(2,4)  = -1.5*C2+0.5*C3
      SMAT(2,5)  = -C7
      SMAT(2,6)  = 2.*C9+C10
      SMAT(2,7)  = PR*(-1.5*A2) + (-1.5*C1+0.5*C3)
```

```
      SMAT(2,8) = -C6
      SMAT(2,9) = PR*A1 + C9
      SMAT(2,10)= -0.5*C3
C
      SMAT(3,1) = DXY*(3.*C4-0.5*C5)
      SMAT(3,2) = DXY*C8
      SMAT(3,3) = DXY*(4.*C11-C12)
      SMAT(3,4) = DXY*(0.5*C5)
      SMAT(3,6) = DXY*C12
      SMAT(3,7) = DXY*(-3.*C4+0.5*C5)
      SMAT(3,8) = DXY*(-C8)
      SMAT(3,9) = DXY*2.*C11
      SMAT(3,10)= DXY*(-0.5*C5)
C
      SMAT(4,1) = PR*(-1.5*C2-0.5*C3)
      SMAT(4,2) = PR*C7
      SMAT(4,3) = PR*(-C10)
      SMAT(4,4) = 1.5*A2 + PR*(1.5*C1+1.5*C2+0.5*C3)
      SMAT(4,5) = PR*(C6+2.*C7)
      SMAT(4,6) = 2.*A1+PR*C10
      SMAT(4,7) = PR*0.5*C3
      SMAT(4,10)= -1.5*A2+PR*(-1.5*C1-0.5*C3)
      SMAT(4,11)= PR*(-C6)
      SMAT(4,12)= A1+PR*(C9)
C
      SMAT(5,1) = -1.5*C2-0.5*C3
      SMAT(5,2) = C7
      SMAT(5,3) = -C10
      SMAT(5,4) = PR*(1.5*A2) +1.5*C1+1.5*C2+0.5*C3
      SMAT(5,5) = C6+2.*C7
      SMAT(5,6) = PR*2.*A1+C10
      SMAT(5,7) = 0.5*C3
      SMAT(5,10)= PR*(-1.5*A2) +(-1.5*C1-0.5*C3)
      SMAT(5,11)= -C6
      SMAT(5,12)= PR*A1+C9
C
      SMAT(6,1) = DXY*(-0.5*C5)
      SMAT(6,3) = DXY*(-C12)
      SMAT(6,4) = DXY*(3.*C4+0.5*C5)
      SMAT(6,5) = DXY*C8
      SMAT(6,6) = DXY*(4.*C11+C12)
      SMAT(6,7) = DXY*0.5*C5
      SMAT(6,10)= DXY*(-3.*C4-0.5*C5)
      SMAT(6,11)= DXY*(-C8)
      SMAT(6,12)= DXY*2.*C11
C
      SMAT(7,1) = -1.5*A2 +PR*(-1.5*C1-0.5*C3)
      SMAT(7,2) = PR*C6
      SMAT(7,3) = -A1-PR*C9
      SMAT(7,4) = PR*0.5*C3
      SMAT(7,6) = -PR*C9
      SMAT(7,7) = 1.5*A2 +PR*(1.5*C1+1.5*C2+0.5*C3)
```

```
      SMAT(7,8) = PR*(-C6-2.*C7)
      SMAT(7,9) = -2.*A1+PR*(-2.*C9-C10)
      SMAT(7,10)= PR*(-1.5*C2-0.5*C3)
      SMAT(7,11)= -PR*C7
      SMAT(7,12)= PR*C10
C
      SMAT(8,1) = PR*(-1.5*A2) -1.5*C1-0.5*C3
      SMAT(8,2) = C6
      SMAT(8,3) = -PR*A1-C9
      SMAT(8,4) = 0.5*C3
      SMAT(8,6) = -C9
      SMAT(8,7) = PR*1.5*A2 +1.5*C1+1.5*C2+0.5*C3
      SMAT(8,8) = -C6-2.*C7
      SMAT(8,9) = PR*(-2.*A1)-2.*C9-C10
      SMAT(8,10)= -1.5*C2-0.5*C3
      SMAT(8,11)= -C7
      SMAT(8,12)= C10
C
      SMAT(9,1) = DXY*(-3.*C4-0.5*C5)
      SMAT(9,2) = DXY*C8
      SMAT(9,3) = DXY*(-2.*C11)
      SMAT(9,4) = DXY*0.5*C5
      SMAT(9,7) = DXY*(3.*C4+0.5*C5)
      SMAT(9,8) = DXY*(-C8)
      SMAT(9,9) = DXY*(-4.*C11-C12)
      SMAT(9,10)= DXY*(-0.5*C5)
      SMAT(9,12)= DXY*C12
C
      SMAT(10,1) = PR*(-0.5*C3)
      SMAT(10,4) = -1.5*A2 +PR*(-1.5*C1+0.5*C3)
      SMAT(10,5) = PR*C6
      SMAT(10,6) = -A1
      SMAT(10,7) = PR*(-1.5*C2+0.5*C3)
      SMAT(10,8) = PR*C7
      SMAT(10,9) = -PR*C10
      SMAT(10,10)= 1.5*A2 +PR*(1.5*C1+1.5*C2-0.5*C3)
      SMAT(10,11)= PR*(-C6+2.*C7)
      SMAT(10,12)= -2.*A1 +PR*(-2.*C9+C10)
C
      SMAT(11,1) = -0.5*C3
      SMAT(11,4) = PR*(-1.5*A2)-1.5*C1+0.5*C3
      SMAT(11,5) = C6
      SMAT(11,6) = -PR*A1
      SMAT(11,7) = -1.5*C2+0.5*C3
      SMAT(11,8) = C7
      SMAT(11,9) = -C10
      SMAT(11,10)= PR*1.5*A2+1.5*C1+1.5*C2-0.5*C3
      SMAT(11,11)= -C6+2.*C7
      SMAT(11,12)= PR*(-2.*A1) -2.*C9+C10
C
      SMAT(12,1) = DXY*(-0.5*C5)
      SMAT(12,4) = DXY*(-3.*C4+0.5*C5)
```

```
      SMAT(12,5) = DXY*C8
      SMAT(12,6) = DXY*(-2.*C11)
      SMAT(12,7) = DXY*0.5*C5
      SMAT(12,9) = DXY*(-C12)
      SMAT(12,10)= DXY*(3.*C4-0.5*C5)
      SMAT(12,11)= DXY*(-C8)
      SMAT(12,12)= DXY*(-4.*C11+C12)
C
      CALL SMULT(SMAT,D,SMAT,12,12)
C
      RETURN
      END
C
C
C:-----------------------------------------------------
C: SUBROUTINE SUBTRC WILL SUBTRACT THE N X M MATRIX [A]
C  AND N X M MATRIX :
C:-----------------------------------------------------
C
         SUBROUTINE SUBTRC (A,B,C,N,M)
         IMPLICIT DOUBLE PRECISION (A-H,O-Z)
         DIMENSION A(N,M),B(N,M),C(N,M)
C
         DO 10 I = 1,N
         DO 10 J = 1,M
  10      C(I,J) = A(I,J) - B(I,J)
         RETURN
         END
C
C
C-------------------------------------------------------
C SUBROUTINE TEMPER CALCULATE THE RESULTING GLOBAL NODAL :
C FORCES CAUSED BY THE TEMPERATURE  DIFFERENTIAL         :
C BETWEEN THE TOP AND THE BOTTOM OF SLAB AND STORE IN    :
C MATRIX [TFT].  HERMAL MOMENT PER UNIT LENGTH (BB) IS   :
C ALSO PASSED BACK TO THE MAIN PROGRAM.                  :
C-------------------------------------------------------
C
        SUBROUTINE TEMPER (TFT,IND1,IND3,XCOOR,YCOOR,BB)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    DIMENSION IND1(NOE),IND3(NOE),XCOOR(NNOD),YCOOR(NNOD)
        DIMENSION TFT(NDISP)
        INTEGER BW,EN
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
     COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
     +                 NOE,NDISP,NNOD,BW,NBK,IRUN
C
C--------------------------------------------------------
C READ IN COEFFICIENT OF THERMAL EXPANSION AND TEMPERATURE:
C READ IN COEFFICIENT OF TRHEMAL EXPANSION AND TEMPERATURE:
C AT THE TOP AND BOTTOM OF THE SLAB.                      :
C--------------------------------------------------------
```

```
          READ(3,*) ALPHA,TEMT,TEMB
          WRITE(4,1000) ALPHA,TEMT,TEMB
          DELAT=TEMB-TEMT
          BB=(E/(1.-PR))*ALPHA*DELAT*T**2./12.
        COT=1./TAN(ANG)
        IF(ANGLE.EQ.90.) COT=0.0
        SINA = SIN(ANG)
C
          DO 10 EN=1,NOE
            ELEN = XCOOR(IND3(EN)) - XCOOR(IND1(EN))
            EWIDTH = YCOOR(IND1(EN)+1) - YCOOR(IND1(EN))
C
            II = IND1(EN) * 3 - 2
            JJ = II + 3
            KK = IND3(EN) * 3 - 2
            LL = KK + 3
C
          F1 = BB * 2.*COT
            F2 = BB * ELEN /(2.*SINA)
            F3 = BB * EWIDTH /(2.*SINA*SINA)
          TFT(II) = TFT(II)+F1
            TFT(II+1)=TFT(II+1)-F2
            TFT(II+2)=TFT(II+2)+F3
          TFT(JJ) = TFT(JJ)-F1
            TFT(JJ+1)=TFT(JJ+1)+F2
            TFT(JJ+2)=TFT(JJ+2)+F3
          TFT(KK) = TFT(KK)-F1
            TFT(KK+1)=TFT(KK+1)-F2
            TFT(KK+2)=TFT(KK+2)-F3
          TFT(LL) = TFT(LL)+F1
            TFT(LL+1)=TFT(LL+1)+F2
            TFT(LL+2)=TFT(LL+2)-F3
   10     CONTINUE
 1000 FORMAT(' COEFF. OF THERMAL EXPANSION  TEMP AT TOP',
     +'  TEMPERATURE AT BOTTOM',/,F10.8,10X,F6.2,2X,F6.2)
        RETURN
        END
C
C
C-----------------------------------------------------------
C SUBROUTINE TEMPFM ADD MOMENT PER UNIT LENGTH CAUSED      |
C BY TEMP. DIFFERENTIALS TO  ELEMENTAL STRESS-COUPLES      |
C [ELMON]                                                  |
C-----------------------------------------------------------
C
      SUBROUTINE TEMPFM(ELMON,BB)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION ELMON(12)
      INTEGER BW
       COMMON /DATAR/ T,E,PR,SMOD,A,B,ANGLE,ANG
   COMMON /DATAI/ NXDIV1,NXDIV2,NXDIV,NYDIV,NXNOD,NYNOD,
     +                     NOE,NDISP,NNOD,BW,NBK,IRUN
```

```
C
      COT = 1./TAN(ANG)
      IF(ANGLE.EQ.90.) COT=0.0
      SINA= SIN(ANG)
C
        BB0=0.
      BB1=BB / SINA
      BB2=BB /(SINA*SINA)
C
      ELMON(1)=ELMON(1)-BB1
      ELMON(2)=ELMON(2)-BB2
      ELMON(3)=ELMON(3)-BB0
      ELMON(4)=ELMON(4)-BB1
      ELMON(5)=ELMON(5)-BB2
      ELMON(6)=ELMON(6)-BB0
      ELMON(7)=ELMON(7)-BB1
      ELMON(8)=ELMON(8)-BB2
      ELMON(9)=ELMON(9)-BB0
      ELMON(10)=ELMON(10)-BB1
      ELMON(11)=ELMON(11)-BB2
      ELMON(12)=ELMON(12)-BB0
      RETURN
      END
C
C
C-------------------------------------------------------
C|  SUBROUTINE XTRACT WILL XTRACT THE 12 X 12 ELEMENTAL |
C|  DISP.VECTOR {EDELTA} FROM THE NDISP X 1 STRUCTURAL  |
C|  DISPLACEMENT VECTOR {DELTA}.                        |
C-------------------------------------------------------
C
        SUBROUTINE XTRACT (DELTA,EDELTA,INDEX,NDISP)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION DELTA(NDISP),EDELTA(12)
        DIMENSION INDEX(12)
C
        DO 30 I =1,12
          II = INDEX(I)
          EDELTA(I) = DELTA(II)
   30   CONTINUE
        RETURN
        END
C
C
C----------------------------------------------------
C|  SUBROUTINE ZERO WILL INITIALIZE THE M X N MATRIX
C   [A] EQUAL TO ZERO. |
C----------------------------------------------------
C
        SUBROUTINE ZERO(A,M,N)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION A(M,N)
```

```
      DO 20 I=1,M
         DO 10 J=1,N
            A(I,J) = 0.0
10       CONTINUE
20    CONTINUE
      RETURN
      END
```

APPENDIX G

A LISTING OF THE INPLANE II PROGRAM

```
C=========================================================
C THIS PROGRAM IS TO ANALYZE  INPLANE EFFECTS OF PAVEMENT
C SYSTEM SUBJECTED TO THERMAL GRADIENTS, SHRIN., FRICTION
C ,WHICH IS COMPOSED OF INPLANE PARALLELOGRAM ELEMENTS.
C
C     NEW STIFFNESS MATRIX.
C     CHANGE LOADING SYSTEM. --------- FRICTION LOADING.
C     SUBROUTINE MAX
C     LIMIT OF THIS PROGRAM.
C              NUMBER OF X - DIVISION : 40
C              NUMBER OF Y - DIVISION : 10
C                                       April 5. 1987
C                         SPRING  1987.    BYUNG-WAN  JO
C=========================================================
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER TNODE,TNEQ
DIMENSION DISP(1386,1),BM(3,8),TLOAD(1386,1),SHLOD(1386,1),
+PLOAD(1386,1),JTDOF(4,11),JTNODE(4,11),XJC(4,11),
TRCLD(1386,1),XJC2(4,11),FCONS(3,3),CONS(3,3),STRS(4,3),
ETHRM(3,1),SS(3,1),STRN(3,1),KKV(8),QL(8,1),SUMX(250),
SUMY(250),XMIN(250),XMAX(250),YMAX(250),YMIN(250),NDSM(250)
      COMMON/TEMPF/TO,TT,TM,TB,THK,ALPHA
 COMMON/GRID/YCRD(11),XCRD(3,41),YMID,XMID(3),GAP(2),TNODE,
NYC,NXC(3),INDEX(600,4),IDOF(693,2),NTOTEL,KVOID(100),NVD
      COMMON/PROP/EC,POI,ESHR,PHI,SGSTF
      COMMON/BLOC1/ELAS(3,3),RG(2)
      COMMON/BLOC2/STIF(1000,26),STIF2(1000,26)
      COMMON/PLATE/QPL(2079,1)
      CHARACTER *50 TITLE,COMND
      CHARACTER *15 FILE1,FILE2,FILE7
       WRITE (*,1001)
 1001 FORMAT(' ENTER NAME OF INPUT FILE:')
       READ(*,1000) FILE1
       WRITE(*,1002)
 1002 FORMAT(' ENTER NAME OF FILE CONTAINING PLATE DISP:')
      READ(*,1000) FILE7
      WRITE(*,1003)
 1003 FORMAT( 'ENTER NAME OF OUTPUT FILE:')
      READ(*,1000) FILE2
      OPEN(1,FILE=FILE1,STATUS='OLD')
      OPEN(2,FILE=FILE2,STATUS='NEW')
      OPEN(7,FILE=FILE7,STATUS='OLD')
      REWIND 1
      REWIND 2
      REWIND 7
C--------------------------------------------------------
C VARIABLE  TYPE    DESCRIPTION
C -----------------------------------------------------
C ------------------------------------------------------
C ALPHA    I      COEFFICIENT OF THERMAL EXPANSION
C EC       R      YOUNG'S MODULUS FOR CONCRETE (KSI)
```

```
C ESHR      R       CONCRETE SHRINKAGE STRAIN
C NBW       I       HALF-BANDWIDTH OF STRUCTURAL STIFFNESS
C NSLB      I       NUMBER OF SLABS IN SYSTEM
C NTOTEL    I       TOTAL NUMBER OF ELEMENTS IN SYSTEM
C NVD       I       NUMBER OF ELEMENTS WITH VOIDS UNDERNEATH
C NYC       I       NUMBER OF NODES IN Y-DIRECTION
C PHI       R       COEFFICIENT OF FRICTION BETWEEN SLAB AND
C                   SUBGRADE
C POI       R       POISSON'S RATIO FOR CONCRETE
C SGSTF     R       SUBGRADE STIFFNESS (KCI)
C TO        R       REFERENCE TEMPERATURE
C TT        R       TEMPERATURE AT TOP OF SLAB
C TM        R       TEMPERATURE AT MIDPOINT OF SLAB THICKNESS
C TB        R       TEMPERATURE AT BOTTOM OF SLAB
C THK       R       THICKNESS OF SLAB
C TNODE     I       TOTAL NUMBER OF NODES IN SYSTEM
C TNEQ      I       TOTAL NUMBER OF DOF'S IN SYSTEM
C--------------------------------------------------------------
C MATRIX  TYPE    DESCRIPTION
C--------------------------------------------------------------
C-------------------------------------------------------------
C BM       R     DERIVATIVES OF SHAPE FUNCTIONS
C CONS     R     CONSTANTS FOR STRESS INTERPOLATION FUNCTIONS
C DISP     R     NODAL DISPLACEMENTS
C ELAS     R     ELASTICITY MATRIX
C ETHRM    R     THERMAL STRAINS
C FCONS    R  CONSTANTS TO SOLVE LEAS/ SQUARES FIT EQUATIONS
C IDOF     I     INDEX OF NODAL DOF'S
C INDEX    I     INDEX OF ELEMENT NODES
C JTDOF    I     INDEX OF JOINT DOF'S
C JTNODE   I     INDEX OF JOINT NODES
C KKV      I  INDEX FOR CONVERTING STRUCTURAL DOF'S TO LOCAL
C KVOID    I     ELEMENTS WITH VOIDS UNDERNEATH
C NXC      I    NUMBER OF NODES IN X-DIRECTION FOR EACH SLAB
C PLOAD    R     LOADING ARRAY
C QL       R     ELEMENT NODAL DISPLACEMENTS
C QPL      R     PLATE BENDING DISPLACEMENTS
C RG       R     GAUSS POINTS
C SS       R     NODAL STRESSES
C STRN     R     ELEMENT STRAINS
C STRS     R     GAUSS POINT STRESSES FOR SOLUTION OF LEAST
C                SQUARES FIT
C STIF     R     STRUCTURAL STIFFNESS MATRIX
C STIF2    R     STRUCTURAL STIFFNESS MATRIX
C SUMX     R  SUM OF NODAL STRESSES IN X-DIR FOR TAKING AVGS
C SUMY     R  SUM OF NODAL STRESSES IN Y-DIR FOR TAKING AVGS
C TLOAD    R     LOADING ARRAY
C XCRD     R     X-COORDINATES OF NODES
C XJC      R     X-COORDINATES OF JOINTS (REFERENCE)
C XJC2     R     X-COORDINATES OF JOINTS (CHECK)
C YCRD     R     Y-COORDINATES OF NODES
C-------------------------------------------------------------
```

```
C
C---BEGIN READING INPUT-------------------------------------
      READ(1,*) ITER
      DO 999 KITER=1,ITER
      READ (1,1000) TITLE
      READ (1,*) EC,POI,ESHR
      READ (1,*) NSLB,ANGLE
      READ (1,*) NYC
      READ (1,*) (NXC(J),J=1,NSLB)
      READ (1,*) (YCRD(J),J=1,NYC)
      DO 5 I=1,NSLB
      READ (1,*) (XCRD(I,J),J=1,NXC(I))
 5    CONTINUE
      IF (NSLB .GT. 1) THEN
      READ (1,*) (GAP(J),J=1,NSLB-1)
      END IF
      READ (1,*) NVD
      IF( NVD .EQ. 0)GOTO 6
      READ (1,*) (KVOID(I), I=1,NVD)
 6    READ (1,*) PHI,SGSTF
      READ (1,*) TO,TT,TM,TB,THK,ALPHA
      READ(1,1000) COMND(1:8)
      IF (NSLB .GT. 1) THEN
      II = 2
 7     DO 8 IA=II,NSLB
        DO 8 JA=1,NXC(IA)
          XCRD(IA,JA)=XCRD(IA,JA)+GAP(II-1)
 8     CONTINUE
         IF ( NSLB .EQ. 3 .AND. II .EQ. 2) THEN
          II = 3
          GOTO 7
         END IF
      END IF
      KEL=0
      KNODE=0
C--------------------------------
C     CHANGE SKEWED ANGLE TO RADIAN.
C------------------------------------
      PI=4.0*ATAN(1.0)
      ANG=ANGLE*PI/180.
      COT=1./DTAN(ANG)
      IF(ANGLE.EQ.90.0) COT=0.0
C--------------------------------------------------------
C  FORM ELEMENT COMPOSITION INDEX MATRIX. INDEX(KEL,J) IS
C  NODE NO. CORRESP. to LOCAL NODE POSITION J FOR ELEMENT
C  NUMBER KEL.
C--------------------------------------------------------
      DO 10 KSLB=1,NSLB
      DO 20 I=1,NXC(KSLB)-1
      KNODE=KNODE+1
      DO 30 J=1,NYC-1
      KEL = KEL +1
```

```
      INDEX(KEL,1)=KNODE
      INDEX(KEL,2)=KNODE+1
      INDEX(KEL,3)=KNODE+NYC
      INDEX(KEL,4)=KNODE+NYC+1
      KNODE=KNODE+1
 30   CONTINUE
 20   CONTINUE
C------INCREMENT NODE COUNTER FOR JOINT----------
      KNODE=KNODE+NYC
 10   CONTINUE
C----------------------------------------------------------
C SUM TOTAL NUMBER OF ELEMENTS TOATAL NUMBER OF NODES AND
C DETERMINE THE HALF-BAND WIDTH
C----------------------------------------------------------
      NBW=2*NYC+4
      NTOTEL=0
      DO 40 I=1,NSLB
      NTOTEL= NTOTEL+(NYC-1)*(NXC(I)-1)
 40   CONTINUE
      KEL=0
      TNODE=0
      DO 50 I=1,NSLB
      TNODE=TNODE+NYC*NXC(I)
 50   CONTINUE
C--------------------------------------------------
C  READ PLATE BENDING DISPLACEMENTS
C--------------------------------------------------
      IF(COMND(1:4) .EQ. 'READ') THEN
      READ(7,166)
  166 FORMAT(///////////)
      DO 15 IQR=1,3*TNODE-2,3
      READ(7,16) (QPL(IQRR,1),IQRR=IQR,IQR+2)
 15   CONTINUE
 16   FORMAT(5X,3(4X,D14.8))
      READ(1,1000) COMND(1:22)
      END IF
C-----------------------------------------------------------
C  FORMF DOF INDEX MATRIX. IDOF(I,J) FOR J=1,8 CONTAINS THE
C  GLOBAL DOF CORRESP. TO LOCAL DOF. USED FOR INSERTION OF
C  STIFFNESS COEFF. INTO GLOBAL BANDED STIFFNESS MATRIX.
C-----------------------------------------------------------
      IF(COMND(1:4) .EQ. 'SKIP') GOTO 999
      KND=0
      KDOF=0
C-------------------------------------------------------
C  DETERMINE Y-COORDINATE OF LONGITUDINAL CENTERLINE
C-------------------------------------------------------
      YMID=(YCRD(NYC)-YCRD(1))/2.0
C
      DO 62 J=1,NSLB
C------------------------------------------------------
C  DETERMINE X-COORDINATE OF LATERAL CENTERLINE
```

```
C-----------------------------------------------------
      XMID(J)=XCRD(J,1)+(XCRD(J,NXC(J))-XCRD(J,1))/2.0
      XMIDT=XMID(J)
C
      DO 64 K=1,NXC(J)
      CHKX=XCRD(J,K)-XMID(J)
C
      DO 66 L=1,NYC
      CHKY=YCRD(L)-YMID
C
      IF(DABS(CHKX) .LT. 0.01 ) THEN
         IF( DABS(CHKY) .LT. 0.01) THEN
           KND=KND+1
           IDOF(KND,1)=0
           IDOF(KND,2)=0
         ELSE
           KND=KND+1
           IDOF(KND,1)=0
           KDOF=KDOF+1
           IDOF(KND,2)=KDOF
         END IF
      ELSE
         KND=KND+1
         KDOF=KDOF+1
         IDOF(KND,1)=KDOF
         KDOF=KDOF+1
         IDOF(KND,2)=KDOF
      END IF
 66   CONTINUE
 64   CONTINUE
 62   CONTINUE
C-----------------------------------------------
C   DETERMINE TOTAL NUMBER OF DOF'S
C-----------------------------------------------
      TNEQ=IDOF(TNODE,2)
C----------------------------------------------
C   ECHO INPUT
C----------------------------------------------
      WRITE(2,900)
 900  FORMAT( //)
      WRITE(2,*) TITLE
      WRITE(*,*) TITLE
      WRITE (2,800) NSLB
 800  FORMAT (5X,'NUMBER OF SLABS = ',I3)
      WRITE(2,790) ANGLE
 790  FORMAT(5X,'SKEWED ANGLE IN DEGREE = ',F8.2)
      DO 600 I=1,NSLB
 600  WRITE(2,801) I,NXC(I)
 801  FORMAT(5X,'# OF X COORDINATES FOR SLAB ',I2,' = ',I3)
      WRITE(2,802) NYC
 802  FORMAT(5X,'NUMBER OF YCOORDINATES =',I3)
      WRITE(2,803) EC
```

```
 803  FORMAT(5X,'YOUNG`S MODULUS FOR CONCRETE = ',F8.2)
C
      DO 708 J=1,NSLB-1
      WRITE(2,709) J,GAP(J)
 709  FORMAT(5X,'GAP (',I2,' ) = ',F10.6)
 708  CONTINUE
      WRITE(2,804) POI
 804  FORMAT(5X,'POISSON`S RATIO = ',F5.3)
      WRITE(2,807) THK
 807  FORMAT(5X,'SLAB THICKNESS = ',F5.3)
      WRITE(2,805) ESHR
 805  FORMAT(5X,'SHRINKAGE STRAIN = ',E10.5)
      WRITE(2,806) TO,TT,TM,TB
806  FORMAT(5X,'T GRADIENT: TO=',F7.3,'TT=',F7.3,'TM= ',
     + F7.3,'  TB= ',F7.3)
      WRITE(2,808) ALPHA
808  FORMAT(5X,'COEFF. OF THERMAL EXPANSION = ',E10.4)
      WRITE(2,809) SGSTF
 809  FORMAT(5X,'SUBGRADE STIFFNESS = ',E10.4)
      WRITE(2,810) PHI
 810  FORMAT(5X,'COEFFICIENT OF FRICTION = ',F6.3)
      IF(NVD .EQ. 0)GOTO 601
      KK=0
      WRITE(2,811)
 811  FORMAT(5X,'ELEMENTS WITH VOIDS UNDERNEATH:')
      KV=NVD/17+1
      DO 602 KM=1,KV
      WRITE(2,812) (KVOID(KK+J),J=1,17)
      KK=KK+17
 602  CONTINUE
 812  FORMAT(5X,17I4)
C----------------------------------------------------
C   OPTIONAL OUTPUT FOR NODAL DEGREES OF FREEDOM
C----------------------------------------------------
 601  IF(COMND(1:8) .EQ. 'NO PRINT') THEN
      READ (1,1000) COMND(1:22)
      GOTO 620
      END IF
      WRITE(2,821) TNEQ
 821  FORMAT(5X,'NODAL DEGREES OF FREEDOM: TOTAL DOFS =',I4)
      WRITE(2,822)
 822  FORMAT(5X,///,'NODE NO.',9X,'X-AXIS','Y-AXIS')
      DO 615 II=1,TNODE
 615  WRITE(2,823) II,(IDOF(II,JJ),JJ=1,2)
 823  FORMAT(8X,I3,8X,2I6)
C
C------------------------------------------------------
C  FORM E. MATRIX [ELAS] AND ARRAY OF GAUSS POINTS [RG]
C------------------------------------------------------
 620  CALL ZERO(ELAS,3,3)
      CALL ZERO(DISP,TNEQ,1)
      ELAS(1,1)=1.0
```

```
      ELAS(1,2)=POI
      ELAS(2,1)=POI
      ELAS(2,2)=1.0
      ELAS(3,3)=(1-POI)/2.0
      EE=EC/(1-POI**2)
      CALL SMULT(EE,ELAS,3,3)
         RG(1)=-1.*SQRT(1.0/3.0)
         RG(2)= SQRT(1.0/3.0)
C-----------------------------------------------
C  FORMATION OF BANDED STIFFNESS MATRIX [STIF]
C-----------------------------------------------
        CALL FORMST(TNEQ,NBW,THK,ANGLE,ANG)
        CALL SYMSOL(PLOAD,TNEQ,NBW,1)
C------------------------------------------------------
C  FORM INDEX MATRIX OF NODES AND DOF'S ALONG JOINT(S)
C  FOR TEMPERATURE ITERATIONS TO CLOSING.
C------------------------------------------------------
        IF ( NSLB .EQ. 1) GOTO 1200
        CALL JOINT(JTDOF,JTNODE,XJC,ANGLE,ANG)
        WRITE(2,632)
 632    FORMAT(/,' JOINT DOFS ')
        DO 630 IPJ=1,2*(NSLB-1)
 630       WRITE(2,631)IPJ,(JTDOF(IPJ,IPPJ),IPPJ=1,NYC)
 631    FORMAT(10I5)
C--------------------------------------------------------
C  FORM SHR. AND TRACTION FORCE VECTORS AND SOLVE FOR THE
C  DISPLACEMENTS
C--------------------------------------------------------
 1200   IF (ESHR .NE. 0.0) THEN
   CALL SHLODE (PLOAD,SHLOD,TRCLD,TNEQ,NBW,THK,ANGLE,ANG)
        CALL SYMSOL(PLOAD,TNEQ,NBW,2)
        CALL COPY(PLOAD,DISP,TNEQ,1)
C--------------------------------------------------------
C ADJUST JOINT COORD.  FOR DISPLACEMENTS DUE TO SHRINKAGE
C     AND TRACTION
C--------------------------------------------------------
      IF ( NSLB .EQ. 1) GOTO 1201
      DO 90 J=1,2*(NSLB-1)
      DO 90 K=1,NYC
         L=JTDOF(J,K)
         XJC(J,K)=XJC(J,K)+PLOAD(L,1)
 90   CONTINUE
      END IF
C-------------------------------------------------------
C APPLY FULL THERMAL LOADING TO DETERMINE IF JOINTS WILL
C CLOSE DUE TO THERMAL EFFECTS
C-------------------------------------------------------
 1201 IF( TO .EQ. 0.0) GOTO 500
      FR=1.0
      CALL TLODE(FR,PLOAD,SHLOD,TRCLD,TNEQ,ANGLE,ANG)
      CALL COPY(PLOAD,TLOAD,TNEQ,1)
C---------------------------------------------------------
```

```
C   COMPARE SHRINKAGE PLUS THERMAL LOAD WITH FRICTION LOAD.
C---------------------------------------------------------
      DO 103 J=1,TNEQ
        IF(DABS(SHLOD(J,1)).LT.DABS(TRCLD(J,1))) THEN
              TV=PLOAD(J,1) + SHLOD(J,1)
C
           IF(DABS(TV).GT.DABS(TRCLD(J,1))) THEN
C
              IF(DABS(PLOAD(J,1)).GT.DABS(SHLOD(J,1))) THEN
                 TRCLD(J,1)= -1.*TRCLD(J,1)
              END IF
C
              PLOAD(J,1)=PLOAD(J,1)+TRCLD(J,1)+SHLOD(J,1)
           ELSE
              PLOAD(J,1)=0.0
           END IF
        END IF
 103    CONTINUE
C---------------------------------
C
      CALL SYMSOL(PLOAD,TNEQ,NBW,2)
       IF (NSLB .EQ. 1) GOTO 100
      DO 110 J=1,2*(NSLB-1)
      DO 110 K=1,NYC
         L=JTDOF(J,K)
         XJC2(J,K)=XJC(J,K)+PLOAD(L,1)
 110  CONTINUE
      JCF=0
      DO 115 J=1,NSLB-1
         L=2*J-1
         DO 120 K=1,NYC
            DX=XJC2(L+1,K)-XJC2(L,K)
            IF ( DX .LT. 0.0) THEN
              JCF=1
            END IF
 120     CONTINUE
 115  CONTINUE
      IF (JCF .EQ. 1 ) GOTO 150
      CALL COPY (XJC2,XJC,4,11)
C--------------------------------------------------------
C  IF PROGRAM REACHES THIS POINT ,JOINTS HAVE NOT CLOSED
C  SO PROCEED TO OBTAIN ELEMENT STRESSES
C-------------------------------------------------------
 100   CALL ADD(DISP,PLOAD,DISP,TNEQ,1)
      GOTO 500
C--------------------------------------------------------
C  BEGIN ITERATION AND SOLUTION OF FORCE DUE TO T. EFFECTS
C--------------------------------------------------------
 150  CALL THERM(SHLOD,TRCLD,JTDOF,XJC,XJC2,TNEQ,NBW)
C--------------------------------------------------------
C  BEGIN PRINTING OF FINAL OUTPUT
C--------------------------------------------------------
```

```
 500  WRITE(2,1300)
 1300 FORMAT(///,' **** DISPLACED JOINT COORDINATES ***')
      WRITE(2,1310)
 1310 FORMAT(/,'    NODE ',' DOF ','    X-COORD ')
      DO 1350 IP=1,2*(NSLB-1)
       DO 1350 JP=1,NYC
        WRITE(2,1355) JTNODE(IP,JP),JTDOF(IP,JP),XJC(IP,JP)
 1350  CONTINUE
 1355  FORMAT(1X,2I6,2X,F12.5)
      DO 1360 JZ=1,TNODE
      SUMX(JZ)=0.0
      SUMY(JZ)=0.0
      NDSM(JZ)=0
 1360 CONTINUE
      IF (COMND(1:9) .EQ. 'NO STRESS') GOTO 999
      WRITE(2,850)
 850  FORMAT(//79('=')/' ',IN-PLANE NODAL STRESSES IN KSI.'
     +,/,79('=')//' ',
     +'NODE #','   X-AVG    ','    Y-AVG    ','  XMAX    ',
     +'    XMIN    ','YMAX',' YMIN    ',//79('-')//)
C-----------------------------------------------------------
C BEGIN SOLVING FOR ELEMENT STRESS. STRAINS ARE FIRST
C DETERMINED FOR GAUSS POINTS AND STRESSES ARE DETERMINED.
C A LINEAR EQ. IN X AND Y IS THEN FITTED TO STRESSES. THE
C FUNCTION IS THEN USED TO OBTAIN THE NODAL STRESSES.
C-----------------------------------------------------------
      CALL ZERO(FCONS,3,3)
      FCONS(1,1)=4.0
      FCONS(1,2)=2.0*RG(1)+2.0*RG(2)
      FCONS(1,3)=FCONS(1,2)
      RR1=RG(1)**2
      RR2=RG(2)**2
      FCONS(2,1)=2*(RR1+RR2)
      FCONS(2,2)=RR1+RR2+2*RG(1)*RG(2)
      FCONS(3,1)=FCONS(2,1)
      CALL SOLVE(FCONS,SS,3,3,1,1)
      IF (NSLB .EQ. 1) THEN
       KEND=NXC(1)/2+2
       NSUB=0
       KEL=0
       KSLB=1
      ELSE
       KEND=NXC(2)/2+2
       KEL =(NXC(1)-1)*(NYC-1)
       KSLB=2
       NSUB=NXC(1)*NYC
      END IF
      DO 510 KTX=2,KEND
         TA=XCRD(KSLB,KTX)-XCRD(KSLB,KTX-1)
       A=TA/2.0
         DO 520 KTY=2,NYC
            TBB=YCRD(KTY)-YCRD(KTY-1)
```

```
         B=TBB/(2.0*SIN(ANG))
           KEL=KEL+1
C-----------------------------------------------------------
C  FORM ARRAY OF NODAL DISPLACEMENTS [QL] FOR ELEMENT KEL
C-----------------------------------------------------------
                CALL ZERO(QL,8,1)
               DO 540 KQL=1,4
                 M=INDEX(KEL,KQL)
                 MM=IDOF(M,1)
                 LL=IDOF(M,2)
                 KKV(2*KQL-1)=MM
                 KKV(2*KQL)=LL
                 IF( MM .EQ. 0) THEN
                   QL(2*KQL-1,1)=0.0
                 ELSE
                   QL(2*KQL-1,1)=DISP(MM,1)
                 END IF
                 IF ( LL .EQ. 0) THEN
                   QL(2*KQL,1)=0.0
                 ELSE
                   QL(2*KQL,1)=DISP(LL,1)
                 END IF
 540           CONTINUE
C--------------------------------------------------------
C DETERMINE STRAINS AT EACH GAUSS POINT AND MULT. BY E.
C  MATRIX TO OBTAIN GAUSS POINT STRESSES
C--------------------------------------------------------
            NN=0
            DO 530 IG=1,2
            DO 530 JG=1,2
               NN=NN+1
               CALL BMFUN(BM,RG(IG),RG(JG),A,B,ANGLE,ANG)
               CALL MULT(BM,QL,STRN,3,8,1)
               FTG=1.0
               CALL TGRAD(ETHRM,FTG)
               FTG=1.0/THK
               CALL SMULT(FTG,ETHRM,3,1)
               STRN(1,1)=STRN(1,1)+ESHR-ETHRM(1,1)
               STRN(2,1)=STRN(2,1)+ESHR-ETHRM(2,1)
               CALL MULT(ELAS,STRN,SS,3,3,1)
               DO 550 KST=1,3
               STRS(NN,KST)=SS(KST,1)
 550        CONTINUE
 530     CONTINUE
C------------------------------------------------------
C INSERT STRESSES INTO RIGHT SIDE OF THE EQUATION THAT
C SOLVES FOR THE INTERPOLATION FUNCTION CONSTANTS
C------------------------------------------------------
            CALL ZERO(CONS,3,3)
            DO 560 KCON=1,3
               L=0
               DO 570 MG=1,2
```

```
              DO 570 NG=1,2
                L=L+1
                CONS(1,KCON)=CONS(1,KCON)+STRS(L,KCON)
         CONS(2,KCON)=CONS(2,KCON)+STRS(L,KCON)*RG(MG)
         CONS(3,KCON)=CONS(3,KCON)+STRS(L,KCON)*RG(NG)
 570          CONTINUE
 560        CONTINUE
              CALL SOLVE(FCONS,CONS,3,3,2,3)
C-- NODAL STRESSES USING INTERPOLATION FUNCTION--------
              MND=0
              DO 580 KSS=1,2
              DO 580 LSS=1,2
                  MND=MND+1
C NODAL POINTS COORD. AT EACH CORNER FOR (0,1) INTERVAL.
C                      XL=DBLE(KSS)-1.0
C                      YL=DBLE(LSS)-1.0
C      FOR (-1,1) INTERVAL.
              XL=1.0
            IF(KSS.EQ.1) XL=-1.0
            YL=1.0
            IF(LSS.EQ.1) YL=-1.0
           SS(1,1)=CONS(1,1)+CONS(2,1)*XL+CONS(3,1)*YL
           SS(2,1)=CONS(1,2)+CONS(2,2)*XL+CONS(3,2)*YL
           SS(3,1)=CONS(1,3)+CONS(2,3)*XL+CONS(3,3)*YL
CC******* PRINT STRESSES
            NDE=INDEX(KEL,MND)-NSUB
            NDSM(NDE)=NDSM(NDE)+1
            SUMX(NDE)=SUMX(NDE)+SS(1,1)
            SUMY(NDE)=SUMY(NDE)+SS(2,1)
            IF(NDSM(NDE) .EQ. 1) THEN
              XMAX(NDE)=SS(1,1)
              XMIN(NDE)=SS(1,1)
              YMAX(NDE)=SS(2,1)
              YMIN(NDE)=SS(2,1)
              GOTO 580
            END IF
            IF (ABS(SS(1,1)) .GE. ABS(XMAX(NDE))) THEN
              XMAX(NDE)=SS(1,1)
            ELSE
              IF(ABS(SS(1,1)) .LE. ABS(XMIN(NDE))) THEN
                XMIN(NDE)=SS(1,1)
              END IF
            END IF
            IF (ABS(SS(2,1)) .GE. ABS(YMAX(NDE))) THEN
              YMAX(NDE)=SS(2,1)
            ELSE
              IF(ABS(SS(2,1)) .LE. ABS(YMIN(NDE))) THEN
                YMIN(NDE)=SS(2,1)
              END IF
            END IF
 580          CONTINUE
 520     CONTINUE
```

```
 510  CONTINUE
 1000   FORMAT(A)
        DO 700 JOUT=1,(KEND-1)*NYC
        AVGX=SUMX(JOUT)/REAL(NDSM(JOUT))
        AVGY=SUMY(JOUT)/REAL(NDSM(JOUT))
        JP=JOUT+NSUB
WRITE(2,860) JP,AVGX,AVGY,XMAX(JOUT),XMIN(JOUT),YMAX(JOUT),
     + YMIN(JOUT)
C
 860  FORMAT(I4,2X,6E12.4)
 700   CONTINUE
C
      IF(COMND(15:22) .EQ. 'STRESSES') THEN
      CALL MAX(XMAX,YMAX)
      END IF
 999   CONTINUE
 9500 STOP
      END
C
C
C---------------------------------------------------------
C  ADD TWO MATRICES A AND B DIMENSIONED L X M TO GET
C  MATRIX C.
C---------------------------------------------------------
      SUBROUTINE ADD(A,B,C,L,M)
      DOUBLE PRECISION A(L,M),B(L,M),C(L,M)
      DO 10 I=1,L
      DO 20 J=1,M
      C(I,J)=A(I,J)+B(I,J)
 20   CONTINUE
 10   CONTINUE
      RETURN
      END
C
C----------------------------------------------------------
C  SUBROUTINE TO CALCULATE FUNCTION [Bm].
C  MU= VARIABLE IN LOCAL X-AXIS DIRECTION
C  ET= VARIABLE IN LOCAL Y-AXIS DIRECTION
C  A = LENGTH OF ELEMENT IN LOCAL X-AXIS DIRECTION * 0.5
C  B = LENGTH OF ELEMENT IN LOCAL Y-AXIS DIRECTION * 0.5
C----------------------------------------------------------
      SUBROUTINE BMFUN(BM,MU,ET,A,B,ANGLE,ANG)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 MU
      DIMENSION BM(3,8)
C
C ZERO [Bm] MATRIX
C
      CALL ZERO(BM,3,8)
      COT=1./DTAN(ANG)
      IF(ANGLE.EQ.90.) COT=0.0
      AA=-COT/A
```

```
      BB=1./(B*SIN(ANG))
      A4=1./(4.*A)
C
      BM(1,1)=-A4*(1.-ET)
      BM(1,3)=-A4*(1.+ET)
      BM(1,5)=A4*(1.-ET)
      BM(1,7)=A4*(1.+ET)
      BM(2,2)=0.25*(-AA*(1.-ET)-BB*(1.-MU))
      BM(2,4)=0.25*(-AA*(1.+ET)+BB*(1.-MU))
      BM(2,6)=0.25*( AA*(1.-ET)-BB*(1.+MU))
      BM(2,8)=0.25*( AA*(1.+ET)+BB*(1.+MU))
      BM(3,1)=BM(2,2)
      BM(3,2)=BM(1,1)
      BM(3,3)=BM(2,4)
      BM(3,4)=BM(1,3)
      BM(3,5)=BM(2,6)
      BM(3,6)=BM(1,5)
      BM(3,7)=BM(2,8)
      BM(3,8)=BM(1,7)
      RETURN
      END
C
C-------------------------------------
C     CREATE AN ARRAY OF SHAPE FUNCTION
C     OF IN-PLANE ELEMENT.
C-------------------------------------
      SUBROUTINE CMTFUN(CMT,MU,ET)
      DOUBLE PRECISION CMT(8,2),MU,ET
      CALL ZERO (CMT,8,2)
      CMT(1,1)=0.25*(1.-MU)*(1.-ET)
      CMT(3,1)=0.25*(1.-MU)*(1.+ET)
      CMT(5,1)=0.25*(1.+MU)*(1.-ET)
      CMT(7,1)=0.25*(1.+MU)*(1.+ET)
      CMT(2,2)=0.25*(1.-MU)*(1.-ET)
      CMT(4,2)=0.25*(1.-MU)*(1.+ET)
      CMT(6,2)=0.25*(1.+MU)*(1.-ET)
      CMT(8,2)=0.25*(1.+MU)*(1.+ET)
      RETURN
      END
C
C----------------------------------------------------
C  SUBROUTINE TO COPY MATRIX A, DIMENSIONED I X J, INTO
C  MATRIX B WHICH HAS THE SAME DIMENSIONS
C----------------------------------------------------
      SUBROUTINE COPY (A,B,I,J)
      DOUBLE PRECISION A(I,J),B(I,J)
      DO 10 II=1,I
      DO 10 JJ=1,J
       B(II,JJ)=A(II,JJ)
 10   CONTINUE
      RETURN
      END
```

```
C
C---------------------------------------------------------
C SUBROUTINE TO CALCULATE THE LOCAL STIFFNESS MATRIX
C [SK] = LOCAL STIFFNESS MATRIX
C  T   = SLAB THICKNESS
C  PR  = POISSON'S RATIO
C  B   = ASPECT RATIO OF ELEMENT
C  EC  = YOUNG'S MODULUS FOR CONCRETE
C---------------------------------------------------------
      SUBROUTINE ELSTIF(SK,T,PR,B,EC,ANGLE,ANG,KEL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION SK(8,8)
      CALL ZERO(SK,8,8)
C
      B1=1./B
      B2=B1*B1
      COT=1./TAN(ANG)
      IF(ANGLE.EQ.90.0) COT=0.0
      CSC=1./SIN(ANG)
      CSC2=CSC*CSC
      RMD=(1.-PR)/2.
      D=EC*T*B*SIN(ANG)/(12.*(1.-PR**2))
C
      C1=2.*B2*CSC2
      C2=4.*B2*CSC2
      C3=3.*B1*CSC
      C4=6.*B1*COT*CSC
C
      T1=2.*COT
      T2=T1*COT
      T3=2.*T1
      T4=T1*T1
C
C SINCE ONLY ONE HALF OF LOCAL MATRIX  NEED BE INSERTED
C INTO THE GLOBAL MATRIX, ONLY THE UPPER TRIANGULAR PORTION
C OF [SK] WILL BE CALCULATED
C
      SK(1,1)=4.+RMD*(T4+C2-C4)
      SK(1,2)=PR*(-T3+C3)+RMD*(-T3+C3)
      SK(1,3)=2.+RMD*(T2-C2)
      SK(1,4)=PR*(-T1-C3)+RMD*(-T1+C3)
      SK(1,5)=-4.+RMD*(-T4+C1)
      SK(1,6)=PR*(T3+C3)+RMD*(T3-C3)
      SK(1,7)=-2.+RMD*(-T2-C1+C4)
      SK(1,8)=PR*(T1-C3)+RMD*(T1-C3)
      SK(2,2)=T4+C2-C4+4.*RMD
      SK(2,3)=PR*(-T1+C3)+RMD*(-T1-C3)
      SK(2,4)=T2-C2+2.*RMD
      SK(2,5)=PR*(T3-C3)+RMD*(T3+C3)
      SK(2,6)=-T4+C1-4.*RMD
      SK(2,7)=PR*(T1-C3)+RMD*(T1-C3)
      SK(2,8)=-T2-C1+C4-2.*RMD
```

```
      SK(3,3)=4.+RMD*(T4+C2+C4)
      SK(3,4)=PR*(-T3-C3)+RMD*(-T3-C3)
      SK(3,5)=-2.+RMD*(-T2-C1-C4)
      SK(3,6)=PR*(T1+C3)+RMD*(T1+C3)
      SK(3,7)=SK(1,5)
      SK(3,8)=SK(2,5)
      SK(4,4)=T4+C2+C4+4.*RMD
      SK(4,5)=SK(3,6)
      SK(4,6)=-T2-C1-C4-2.*RMD
      SK(4,7)=SK(1,6)
      SK(4,8)=SK(2,6)
      SK(5,5)=SK(3,3)
      SK(5,6)=SK(3,4)
      SK(5,7)=SK(1,3)
      SK(5,8)=SK(2,3)
      SK(6,6)=SK(4,4)
      SK(6,7)=SK(1,4)
      SK(6,8)=SK(2,4)
      SK(7,7)=SK(1,1)
      SK(7,8)=SK(1,2)
      SK(8,8)=SK(2,2)
C
      CALL SMULT(D,SK,8,8)
      RETURN
      END
C
C--------------------------------------------------------------
C   SUBROUTINE TO ASSEMBLE THE STRUCTURAL STIFFNESS MATRIX
C   [STIF] INTO A BANDED FORMAT
C--------------------------------------------------------------
      SUBROUTINE FORMST(TNEQ,NBW,THK,ANGLE,ANG)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER TNEQ,TNODE
      DIMENSION KKV(8),SK(8,8)
COMMON/GRID/YCRD(11),XCRD(3,41),YMID,XMID(3),GAP1,GAP2,
     +NSLB,NYC,NXC(3),INDEX(600,4),IDOF(693,2),NTOTEL,
      COMMON/PROP/EC,POI,ESHR,PHI,SGSTF
      COMMON/BLOC2/STIF(1000,26),STIF2(1000,26)
      CALL ZERO(STIF,1000,26)
      KEL=0
      DO 70 I=1,NSLB
      DO 80 J=1,NXC(I)-1
       X=XCRD(I,J+1)-XCRD(I,J)
      DO 90 K=1,NYC-1
        KEL = KEL+1
        Y= YCRD(K+1)-YCRD(K)
C--------------------------------------------------------
C  DETERMINE ASPECT RATIO OF ELEMENT,(BETA), THEN
C  CALL SUBROUTINE TO FORM ELEMENT STIFFNESS MATRIX [SK]
C-------------------------------------------------------------
        BETA=Y/(SIN(ANG)*X)
        CALL ELSTIF(SK,THK,POI,BETA,EC,ANGLE,ANG,KEL)
```

```
C-----------------------------------------------------------
C  INSERT [SK] INTO STRUCTURAL STIFFNESS MATRIX, [STIF]
C-----------------------------------------------------------
        DO 100 L=1,4
        LL=INDEX(KEL,L)
        MM=IDOF(LL,1)
        NN=IDOF(LL,2)
        KKV(2*L-1)=MM
        KKV(2*L)=NN
 100    CONTINUE
         DO 102 L=1,8
           NEQ=KKV(L)
           IF(NEQ .EQ. 0) GOTO 102
           DO 101 M=L,8
             NOC=KKV(M)
             IF(NOC .EQ. 0) GOTO 101
             NNC=NOC-NEQ+1
             STIF(NEQ,NNC)=STIF(NEQ,NNC)+SK(L,M)
 101       CONTINUE
 102    CONTINUE
  90  CONTINUE
  80  CONTINUE
  70  CONTINUE
      CALL COPY (STIF,STIF2,1000,26)
      RETURN
      END
C
C-------------------------------------------
C     SUBROUTINE IZERO
C-------------------------------------------
      SUBROUTINE IZERO(I,L,M)
C--- SUBROUTINE TO ZERO OUT AN INTEGER ARRAY  L X M
      DIMENSION I(L,M)
      DO 10 J=1,L
      DO 20 K=1,M
      I(J,K)=0
 20   CONTINUE
 10   CONTINUE
      RETURN
      END
C-----------------------------------------------------
C  SUBROUTINE TO FORM THE INDEX ARRAYS OF NODES ALONG
C  THE JOINTS AND THEIR DOF'S AND X-COORDINATES
C-----------------------------------------------------
        SUBROUTINE JOINT(JTDOF,JTNODE,XJC,ANGLE,ANG)
        IMPLICIT REAL*8 (A-H,O-Z)
CC        INTEGER TNEQ
        INTEGER TNODE
        DIMENSION JTDOF(4,11),JTNODE(4,11),XJC(4,11)
CC     +  ,XJC2(4,11),DISP(TNEQ,1)
COMMON/GRID/YCRD(11),XCRD(3,41),YMID,XMID(3),G1,G2,TNODE,
,NYC,NXC(3),INDEX(600,4),IDOF(693,2),NTOTEL,KVOID(100),NVD
```

```
      JJ=0
      COT=1./DTAN(ANG)
      IF(ANGLE.EQ.90.) COT=0.0
C
      DO 170 J=1,NSLB-1
         JJ=JJ+NYC*NXC(J)
         JK=JJ-NYC+1
         DO 175 K=1,NYC
         JTNODE(2*J-1,K)=JK
         JTDOF(2*J-1,K)=IDOF(JK,1)
         JK=JK+1
 175     CONTINUE
       DO 180 K=1,NYC
         JTNODE(2*J,K)=JK
         JTDOF(2*J,K)=IDOF(JK,1)
         JK=JK+1
C180 CONTINUEC--FORM MATRIX OF X-COORDINATES ALONG JOINTS-
         DO 185 K=1,NYC
            XJC(2*J-1,K)=XCRD(J,NXC(J))+YCRD(K)*COT
            XJC(2*J,K)=XCRD(J+1,1)+YCRD(K)*COT
 185     CONTINUE
 170  CONTINUE
        RETURN
        END
```

```
C------------------------------------------------------------
C  MAX    WILL FIND THE MAXIMUM VALUE OF THE MATRIX [X].
C  MAXIMUM VALUE WILL BE FOUND BETWEEN THE GIVEN INCREMENTS.
C------------------------------------------------------------
C
        SUBROUTINE MAX(X,Y)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION X(250),Y(250)
        DOUBLE PRECISION MAXX,MAXY
        INTEGER NODE(44,3)
        CHARACTER*50 TITLE,DIR1*7,DIR2*7
C
     TITLE = '    MAXIMUM IN-PLANE NODAL STRESSES IN KSI'
        READ (1,*) NUMINC,((NODE(I,J),J=1,3),I=1,NUMINC)
        DO 30 I = 1,NUMINC
          WRITE (2,2010) TITLE,NODE(I,1),NODE(I,2)
C
        MAXX = 0.0
        NX = 0
            DO 10 J = NODE(I,1),NODE(I,2),NODE(I,3)
               NX = NX+1
               IF(ABS(MAXX) .LT. ABS(X(NX))) THEN
                MAXX = X(NX)
                MAXNDX = J
              END IF
  10        CONTINUE
C
        MAXY = 0.0
        NY = 0
            DO 20 J = NODE(I,1),NODE(I,2),NODE(I,3)
               NY = NY+1
               IF(ABS(MAXY) .LT. ABS(Y(NY))) THEN
                MAXY = Y(NY)
                MAXNDY = J
              END IF
  20        CONTINUE
              DIR1 = '  SXX '
              DIR2 = '  SYY  '
            WRITE (2,2020) MAXNDX,DIR1,MAXX
            WRITE (2,2021) MAXNDY,DIR2,MAXY
          WRITE (2,2030)
  30    CONTINUE
2000    FORMAT ('1')
2010    FORMAT (///' ',79('=')/' ',A50,I4,' TO ',I4,/
    +    ' ',79('=')//' ',' NODE  DIRECTION      STRESSES',
    +   //' ',35('-')/)
2020    FORMAT (' ',I4,4X,A7,D18.8)
2021    FORMAT (' ',I4,4X,A7,D18.8)
2030    FORMAT (///)
        RETURN
        END
C
```

```
C-------------------------------------------------------
C  SUBROUTINE MULT WILL MULTIPLY AN LxM MATRIX A TIMES
C  AN MxN MATRIX BE TO GET AN LxN MATRIX C
C-------------------------------------------------------
      SUBROUTINE MULT(A,B,C,L,M,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(L,M),B(M,N),C(L,N)
      DO 10 I=1,L
      DO 10 J=1,N
         C(I,J)=0.0
      DO 10 K=1,M
         C(I,J)=C(I,J)+A(I,K)*B(K,J)
   10 CONTINUE
      RETURN
      END
C
C
C--------------------------------------
C     CREATE AN ARRAY OF SHAPE FUNCTION
C     OF PLATE BENDING ELEMENT.
C--------------------------------------
C
      SUBROUTINE RBFUN(RB,MU,ET,A,B)
      IMPLICIT REAL*8 (A-H,J-Z)
      REAL*8 MU,M1,M2
      DIMENSION RB(2,12)
      C18=1./8.
      M1=1.-MU
      M2=1.+MU
      E1=1.-ET
      E2=1.+ET
C
      DO 10 I=1,2
      RB(I,1)=C18*M1*E1*(2.-MU-ET-MU**2-ET**2)
      RB(I,2)=-C18*B*M1*E2*E1**2
      RB(I,3)=C18*A*M2*E1*M1**2
      RB(I,4)=C18*M1*E2*(2.-MU+ET-MU**2-ET**2)
      RB(I,5)=C18*B*M1*E1*E2**2
      RB(I,6)=C18*A*M2*E2*M1**2
      RB(I,7)=C18*M2*E1*(2.+MU-ET-MU**2-ET**2)
      RB(I,8)=-C18*B*M2*E2*E1**2
      RB(I,9)=-C18*A*M1*E1*M2**2
      RB(I,10)=C18*M2*E2*(2.+MU+ET-MU**2-ET**2)
      RB(I,11)=C18*B*M2*E1*E2**2
      RB(I,12)=-C18*A*M1*E2*M2**2
  10  CONTINUE
      RETURN
      END
C----------------------------------------------------
C  SUBROUTINE TO FORM THE NODAL LOAD VECTOR DUE TO
C  SHRINKAGE AND SUB-GRADE FRICTION
C----------------------------------------------------
```

```
      SUBROUTINE SHLODE(SHLOD,TRCLD,TNEQ,NBW,THK,ANGLE,ANG)
        IMPLICIT REAL*8 (A-H,O-Z)
        INTEGER TNEQ,TNODE
        DIMENSION FSHR(8,1),FTRACT(8,1),SHLOD(TNEQ,1),
     + TRCLD(TNEQ,1),
     + SUMINT(8,1),KKV(8),UNIT(3,1),FTEMP(8,3),CHK(4)
      COMMON/GRID/XCRD(3,41),YMID,XMID(3),G1,G2,TNODE,
     +NSLB,NYC,IDOF(693,2),NTOTEL,KVOID(100),NVD
      COMMON/PLATE/QPL(2079,1)
      COMMON/PROP/EC,POI,ESHR,PHI,SGSTF
      COMMON/BLOC1/ELAS(3,3),RG(2)
C-----------------------------------------------------
C   VARIABLE  TYPE        DESCRIPTION
C-----------------------------------------------------
C   FSHR      RM     ELEMENT FORCES DUE TO SHRINKAGE
C   FTRACT    RM      ELEMENT FORCES DUE TO FRICTION
C   UNIT      RM         UNIT ARRAY
C   BM        RM
C   FTEMP     RM    RRAYS FOR PERFORMING INTEGRATION
C   SUMINT    RM
C-----------------------------------------------------
 50     CALL ZERO(PLOAD,TNEQ,1)
        CALL ZERO(SHLOD,TNEQ,1)
        CALL ZERO(TRCLD,TNEQ,1)
      TFRICT=PHI*SGSTF
      UNIT(1,1)=1.0
      UNIT(2,1)=1.0
      UNIT(3,1)=0.0
      KTV=1
      KEL=0
      DO 110 I=1,NSLB
      DO 110 KTX=2,NXC(I)
C-------------------------------------------------------
C  DETERMINE ELEMENT DIM. FOR SHAPE FUNCTION , A AND B
C--------------------------------------------------------
      TA=XCRD(I,KTX)-XCRD(I,KTX-1)
      A=TA/2.0
      DO 120 J=2,NYC
         TBB=YCRD(J)-YCRD(J-1)
         B=TBB/(2.0*SIN(ANG))
         KEL=KEL+1
         CALL ZERO(FSHR,8,1)
C-----------------------------------
C  PERFORM GAUSS POINT INTEGRATION
C-----------------------------------
                 DO 140 IG=1,2
                 DO 141 JG=1,2
                 CALL BMFUN(BM,RG(IG),RG(JG),A,B,ANGLE,ANG)
                 SCA=-ESHR*THK*A*B*SIN(ANG)
                 CALL SMULT(SCA,BM,3,8)
                 CALL TRANMU(BM,ELAS,FTEMP,8,3,3)
                 CALL MULT(FTEMP,UNIT,SUMINT,8,3,1)
```

```
                 CALL ADD(FSHR,SUMINT,FSHR,8,1)
  141            CONTINUE
  140            CONTINUE
C---------------------------------------------------------
C  DETERMINE WHICH QUADRANTS THE NODES ARE IN AND CALL
C  SUBROUTINE TO FORM ARR. OF NODAL FORCES DUE TO FRICTION
C----------------------------------------------------------
        CHK(1)=XMID(I)-XCRD(I,KTX-1)
        CHK(2)=XMID(I)-XCRD(I,KTX)
        CHK(3)=YMID-YCRD(J-1)
        CHK(4)=YMID-YCRD(J)
        IF( NVD .EQ. 0) THEN
          CALL TRACLD(A,B,FTRACT,KEL,CHK,1,ANG)
           GOTO 161
        ELSE
          IF(KEL .NE. KVOID(KTV)) THEN
            CALL TRACLD(A,B,FTRACT,KEL,CHK,1,ANG)
            GOTO 161
          ELSE
            CALL ZERO(FTRACT,8,1)
            KTV=KTV+1
            GOTO 161
          END IF
        END IF
C---------------------------------------------
C  INSERT FORCES DUE TO SHRINKAGE AND FRICTION
C  INTO STRUCTURAL ARRAY
C---------------------------------------------
 161      DO 145 LDF=1,4
           LLDF=INDEX(KEL,LDF)
           MMDF=IDOF(LLDF,1)
           NNDF=IDOF(LLDF,2)
           KKV(2*LDF-1)=MMDF
           KKV(2*LDF)=NNDF
 145       CONTINUE
          DO 155 KF=1,8
           KKF=KKV(KF)
           IF (KKF .EQ. 0) GOTO 155
TRCLD(KKF,1)=TRCLD(KKF,1)+FTRACT(KF,1)
SHLOD(KKF,1)=SHLOD(KKF,1)+FSHR(KF,1)
 155     CONTINUE
 120  CONTINUE
 110  CONTINUE
C
DO 300 I=1,TNEQ
   IF(DABS(TRCLD(I,1)).GT.DABS(SHLOD(I,1))) THEN
      PLOAD(I,1)=0.0
   ELSE
      PLOAD(I,1)=TRCLD(I,1)+SHLOD(I,1)
   END IF
 300           CONTINUE
C
```

```
        RETURN
        END
C
C
C---------------------------------------------------------
C  MULTIPLY A SCALAR S TIMES A MATRIX [A] WHICH IS
C  DIMENSIONED I X J.
C---------------------------------------------------------
      SUBROUTINE SMULT(S,A,I,J)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(I,J)
      DO 10 L=1,I
      DO 10 M=1,J
       A(L,M)=S*A(L,M)
 10   CONTINUE
      RETURN
      END
C-----------------------------
C       SUBROUTINE SOLVE
C-----------------------------
C
      SUBROUTINE SOLVE(A,B,NN,MM,KK,LL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(NN,MM),B(NN,LL)
      GO TO (1000,2000),KK
 1000 DO 280 N=1,NN
      IF(A(N,1) .GT. 0.0) GOTO 220
      WRITE(*,3001) N,A(N,1)
      WRITE(2,3001) N,A(N,1)
      GOTO 280
 220  DO 260 L=2,MM
         C=A(N,L)/A(N,1)
         I=N+L-1
         IF(NN-I) 260,240,240
 240  J=0
      DO 250 K=L,MM
      J=J+1
 250  A(I,J)=A(I,J)-C*A(N,K)
 260  A(N,L)=C
 280  CONTINUE
      GOTO 500
 2000 DO 450 J=1,LL
      DO 295 N=1,NN
      IF(A(N,1) .NE. 0.0) GO TO 284
      WRITE(*,3001) N,A(N,1)
      WRITE(2,3001) N,A(N,1)
      GO TO 295
 284  DO 285 L=2,MM
      I=N+L-1
      IF(NN-I) 290,285,285
 285  B(I,J)=B(I,J)-A(N,L)*B(N,J)
```

```
 290  B(N,J)=B(N,J)/A(N,1)
 295   CONTINUE
      N=NN
 300  N=N-1
      IF(N) 350,450,350
 350  DO 400 K=2,MM
      L=N+K-1
      IF (NN-L) 400,370,370
 370  B(N,J)=B(N,J)-A(N,K)*B(L,J)
 400  CONTINUE
        GOTO 300
 450  CONTINUE
 500  RETURN
 3001 FORMAT (/,' *** ERROR *** EQUATION # =',I5,3X,
      END
C
C----------------------
C     SUBROUTINE SYMSOL
C----------------------
c
      SUBROUTINE SYMSOL(B,NN,MM,KKK)
C-----A IS THE STIFFNESS MATRIX
C-----B IS THE LOAD VECTOR -----
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION B(NN,1)
      COMMON/BLOC2/A(1000,26),A2(1000,26)
C---- SYMMETRICAL BANDED SOLVER -----
C-----KKK=1 = JUST DECOMPOSITION
C-----KKK=2 = JUST FORWARD AND BACK SUBSTITUTION
      GO TO (1000,2000),KKK
C---- REDUCE MATRIX -----
 1000 continue
      DO 280 N=1,NN
        IF(A(N,1).GT.0.0) GO TO 220
        WRITE (*,3001) N,A(N,1)
        WRITE (2,3001) N,A(N,1)
        GO TO 280
  220   DO 260 L=2,MM
          C = A(N,L)/A(N,1)
          I = N + L - 1
          IF(NN-I) 260,240,240
  240     J = 0
          DO 250 K=L,MM
            J = J + 1
            A(I,J) = A(I,J) - C*A(N,K)
  250       CONTINUE
  260     A(N,L) = C
  280 CONTINUE
      GO TO 500
C---- REDUCE VECTOR (FORWARD SUBSTITUTION) -----
 2000 DO 295 N=1,NN
      IF(A(N,1).NE.0.0) GO TO 284
```

```
      WRITE (*,3001) N,A(N,1)
      WRITE (2,3001) N,A(N,1)
      GO TO 295
  284 DO 285 L=2,MM
      I = N + L - 1
      IF(NN-I) 290,285,285
  285 B(I,1) = B(I,1) - A(N,L)*B(N,1)
  290 B(N,1) = B(N,1)/A(N,1)
  295 CONTINUE
C---- BACK SUBSTITUTION -----
      N = NN
  300 N = N-1
      IF(N) 350,500,350
  350 DO 400 K=2,MM
      L = N + K - 1
      IF(NN-L) 400,370,370
  370  B(N,1) = B(N,1) - A(N,K)*B(L,1)
  400 CONTINUE
      GO TO 300
C
  500 RETURN
C--------------------------------------------------
 3001 FORMAT (/' *** ERROR *** EQUATION # =',I5,3X,
      END
C
C
C------------------------------------------------
C   SUBROUTINE TO PERFORM INTEGRATION OF THERMAL
C   GRADIENT IN Z-DIRECTION
C------------------------------------------------
      SUBROUTINE TGRAD(TSTR,FRACT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION TSTR(3,1)
      COMMON/TEMPF/TO,TT,TM,TB,THK,ALPHA
      A=TM-TO
      C=(TT+TB-2*TM)/6.0
      TF=FRACT*THK*(A+C)
      TSTR(1,1)=TF
      TSTR(2,1)=TF
      TSTR(3,1)=0.0
      CALL SMULT(ALPHA,TSTR,3,1)
      RETURN
      END
C
C-----------------------------------------------------
C  ASSEMBLE AND SOLVE FOR THE FORCE VECTOR DUE TO
C  TEMPERATURE EFFECTS
C-----------------------------------------------------
   SUBROUTINE THERM(SHLOD,TRCLD,JTDOF,XJC,XJC2,TNEQ,NBW)
        IMPLICIT REAL*8 (A-H,O-Z)
        INTEGER TNEQ,TNODE
DIMENSION  DISP(TNEQ,1),JTDOF(4,11),XJC(4,11),XJC2(4,11),
```

```
+  JCLOSE(22,4),PLOAD(1386,1),NFLG(2,11),TLOAD(TNEQ,1),
     +  SHLOD(TNEQ,1),TRCLD(TNEQ,1)
      COMMON/TEMPF/TO,TT,TM,TB,THK,ALPHA
COMMON/GRID/XCRD(3,41),YMID,XMID(3),GAP1,GAP2,TNODE,
     +NSLB,INDEX(600,4),IDOF(693,2),NTOTEL,KVOID(100),NVD
      COMMON/PROP/EC,POI,ESHR,PHI,SGSTF
      COMMON/BLOC1/ELAS(3,3),RG(2)
      COMMON/BLOC2/STIF(1000,26),STIF2(1000,26)
C----------------------------------------------------------
C VARIABLE TYPE   DESCRIPTION
C----------------------------------------------------------
C JCLOSE   IM INFORMATION ON NODES THAT HAVE CLOSED ON ONE
C                 INCREMENT OF THERMAL GRADIENT
C JCLS     I      FLAG TO INDICATE A NODE HAS CLOSED ON AN
C                 INCREMENT
C FF       R      TOTAL FRACTION OF GRADIENT BEING APPLIED
C FLOW     R      LOWER END OF BISECTION INCREMENT
C FHIGH    R      UPPER END OF BISECTION INCREMENT
C STEP     R      INCREMENTAL FRACTION OF THERMAL GRADIENT
C PLOAD    RM     INCREMENTAL LOADING
C TLOAD    RM     TOTAL LOADING
C KCHKT    I    TOTAL NUMBER OF JOINT NODAL PAIRS TO CLOSE
C JCLS     I      COUNTER FOR CLOSED NODAL PAIRS
C KLSCT    I  NUMBER OF NODES TO CLOSE ON A LOAD INCREMENT
C----------------------------------------------------------
      AVG=0.0
      AVG2=0.0
      DEL=0.0
      DEL2=0.0
      NAV=0
      NAV2=0
      DO 100 I=1,NYC
      SUM=SUM+(XJC2(1,I)-XJC2(2,I))/2.0
      AVG=AVG+XJC2(1,I)-XJC(1,I)
 100  CONTINUE
      SUM=SUM/REAL(NYC)
      AVG=AVG/REAL(NYC)
      FF=(AVG-SUM)/AVG
      IF (FF .GT. 1.0) FF=1.0
      AVG=0.0
      SUM=0.0
      KLSCT=0
      CALL IZERO (NFLG,2,11)
      KCHKT=NYC*(NSLB-1)
      JCLS=0
C--------------------------------------------------------
C  INCREMENT FRACTION OF TOTAL APPLIED GRADIENT,FF, AND
C  DETERMINE INCREMENTAL STEP,STEP
C--------------------------------------------------------
 10   STEP=FF-SUM
C--------------------------------------------------------
C  APPLY INCREMENTAL STEP OF THERMAL GRADIENT LOADING TO
```

```
C  SYSTEM AND SOLVE
C--------------------------------------------------------
      CALL COPY(TLOAD,PLOAD,TNEQ,1)
      CALL SMULT(STEP,PLOAD,TNEQ,1)
C------------------------------------------------------
C     COMPARE CURRENT TOTAL THERMAL PLUS SHRINKGE LOAD
C     WITH FRICTION LOAD.
C------------------------------------------------------
C
DO 12 I=1,TNEQ
   TV=SUM*TLOAD(I,1)+SHLOD(I,1)
   IF(DABS(TV).LT.DABS(TRCLD(I,1))) THEN
TV=TV+PLOAD(I,1)
C
   IF(DABS(TV).GT.DABS(TRCLD(I,1))) THEN
C
IF(DABS(PLOAD(J,1)).GT.DABS(SHLOD(J,1))) THEN
                TRCLD(J,1)= -1.*TRCLD(J,1)
        END IF
C
PLOAD(I,1)=TV+TRCLD(I,1)
   ELSE
PLOAD(I,1)=0.0
   END IF
   END IF
   12           CONTINUE
C
      CALL SYMSOL(PLOAD,TNEQ,NBW,2)
      JFLG=0
      KLSCT=0
C---ADD JOINT DISPLACEMENTS AND CHECK FOR CONTACT/OVERLAP-
      DO 15 J=1,NSLB-1
         DO 20 K=1,NYC
C---SKIP FLAGGED NODES THAT HAVE ALREADY CLOSED-----------
           IF (NFLG(J,K) .EQ. 1) GOTO 20
            L=2*J-1
            M=JTDOF(L,K)
            N=JTDOF(L+1,K)
            XJC2(L,K)=XJC(L,K)+PLOAD(M,1)
            XJC2(L+1,K)=XJC(L+1,K)+PLOAD(N,1)
            CHK=XJC2(L+1,K)-XJC2(L,K)
C---IF JOINTS OVERLAP THEN TAKE AVERAGES FOR INTERPOLATION
            IF(CHK .LT. -0.0005) THEN
            JFLG=2
            NAV2=NAV2+1
            DEL2=DEL2+PLOAD(M,1)
            AVG2=AVG2+CHK/2.0
            ELSE
            IF (JFLG .EQ. 2 ) GOTO 20
            IF(CHK .GT. 0.0005) THEN
             AVG=AVG+CHK/2.0
             DEL=DEL+PLOAD(M,1)
```

```
            NAV=NAV+1
C-----------------------------------------------------------
C  IF JOINTS ARE WITHIN TOLERANCE THEN STORE JOINT INDICES
C  TO BE FLAGGED AND STIFFENED IF NO OTHER JOINTS OVERLAP
C-----------------------------------------------------------
           ELSE
              KLSCT=KLSCT+1
              JCLOSE (KLSCT,1)=M
              JCLOSE (KLSCT,2)=N
              JCLOSE (KLSCT,3)=J
              JCLOSE (KLSCT,4)=K
              JFLG=1
            END IF
            END IF
  20     CONTINUE
  15  CONTINUE
C----------------------------------------------------
C  CHECK FLAG TO SEE IF JOINTS HAVE OVERLAPPED,MADE
C  NO CONTACT, OR CLOSED
C----------------------------------------------------
      IF(JFLG .EQ.2) THEN
        CALL IZERO(JCLOSE,22,4)
        AVG2=AVG2/REAL(NAV2)
        DEL2=DEL2/REAL(NAV2)
        FF=FF*(DEL2+AVG2)/DEL2
        NAV2=0
        AVG2=0.0
        DEL2=0.0
        DEL=0.0
        AVG=0.0
        NAV=0
      ELSE
        DEL=DEL/REAL(NAV)
        AVG=AVG/REAL(NAV)
        NAV=0
        SUM=FF
        FF=FF*(DEL+AVG)/DEL
        IF ( FF .GT. 1.0) FF=1.0
        DEL=0.0
        AVG=0.0
        CALL ADD(PLOAD,DISP,DISP,TNEQ,1)
        CALL COPY (XJC2,XJC,4,11)
      END IF
C---FLAG AND STIFFEN CLOSED JOINTS-----------------
      IF (JFLG .EQ. 1) THEN
       DO 25 KK=1,KLSCT
         M=JCLOSE (KK,1)
         N=JCLOSE (KK,2)
         J=JCLOSE (KK,3)
         K=JCLOSE (KK,4)
         NNC=N-M+1
C-----------------------------------------------------
```

```
C  MODIFY STIFFNESS MATRIX TO LOCK CLOSED NODES
C-----------------------------------------------------
          STIF2(M,NNC)=-1000.0*STIF2(M,1)
          STIF2(N,1)=STIF2(N,1)+1000.0*STIF2(M,1)
          STIF2(M,1)=STIF2(M,1)+1000.0*STIF2(M,1)
          NFLG(J,K)=1
          JCLS=JCLS+1
  25  CONTINUE
C--DECOMPOSE NEW STIFFNES MATRIX AND ADD ALL DISPLACEMENTS
C--INTO STORAGE
      CALL COPY(STIF2,STIF,1000,26)
      CALL SYMSOL(PLOAD,TNEQ,NBW,1)
      CALL IZERO(JCLOSE,22,4)
      END IF
C--CHECK TO SEE IF ALL JOINTS HAVE CLOSED AND IF NOT THEN
C  ADD IN NEW
C--JOINT COORDINATES
        IF ( SUM .GE. 0.999) GOTO 999
        IF ( JCLS .EQ. KCHKT) THEN
        STEP=1.0-SUM
        CALL COPY(TLOAD,PLOAD,TNEQ,1)
        CALL SMULT(STEP,PLOAD,TNEQ,1)
C-----------------------------------
C  CHECK AMOUNT OF FRICTION FORCE.
C  COMPARE SHRINKAGE PLUS CURRENT
C  TOTAL AMOUNT OF THERMAL WITH
C  FRICTION LOAD. SUM=1.0
C-----------------------------------
C
DO 200 I=1,TNEQ
   TV=TLOAD(I,1)+SHLOD(I,1)
   IF(DABS(TV).LT.DABS(TRCLD(I,1))) THEN
TV=TV+PLOAD(I,1)
C
   IF(DABS(TV).GT.DABS(TRCLD(I,1))) THEN
C
IF(DABS(PLOAD(J,1)).GT.DABS(SHLOD(J,1))) THEN
                TRCLD(J,1)= -1.*TRCLD(J,1)
        END IF
C
PLOAD(I,1)=TV+TRCLD(I,1)
   ELSE
PLOAD(I,1)=0.0
   END IF
   END IF
  200           CONTINUE
C
C
        CALL SYMSOL(PLOAD,TNEQ,NBW,2)
        CALL ADD(PLOAD,DISP,DISP,TNEQ,1)
        ELSE
        GOTO 10
```

```
      END IF
999   RETURN
      END
C----------------------------------------------------------------
C  SUBROUTINE THAT ASSEMBLES THE THERMAL LOADING MATRIX
C----------------------------------------------------------------
 SUBROUTINE TLODE(FRACT,PLOAD,SHLOD,TRCLD,TNEQ,ANGLE,ANG)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER TNEQ,TNODE
      DIMENSION BM(3,8),F(8,1),PLOAD(TNEQ,1),
     + SHLOD(TNEQ,1),TRCLD(TNEQ,1),
     + SUM(8,1),THGR(3,1),C(8,3),KKV(8),FTRACT(8,1),CHK(4)
      COMMON/GRID/XCRD(3,41),YMID,XMID(3),GAP1,GAP2,TNODE,
     +NSLB,NYC,NXC(3),IDOF(693,2),NTOTEL,KVOID(100),NVD
      COMMON/BLOC1/ELAS(3,3),RG(2)
      COMMON/TEMPF/TO,TT,TM,TB,THK,ALPHA
C--------------------------------------------------------------
C VARIABLE   TYPE    DESCRIPTION
C--------------------------------------------------------------
C BM     RM      DERIVATIVES OF NODAL SHAPE FUNCTIONS
C C      RM      INTERMEDIATE STEP IN INTEGRATION PROCESS
C CHK    RM      CHECKS FOR DIRECTIONS OF FRICTION FORCES
C F      RM      FORCES DUE TO TEMP. AND FRICTION
C FTRACT  RM     ARRAY OF NODAL FORCES DUE TO FRICTION
C SUM    RM      SUM OF GAUSS POINT INTEGRATION STEPS
C---------------------------------------------------------------
C
C---------------------------------------------------------------
C  CALL SUBROUTINE TO PERFORM INTEGRATION IN Z-DIRECTION
C---------------------------------------------------------------
      CALL TGRAD(THGR,FRACT)
      CALL ZERO(PLOAD,TNEQ,1)
      KEL=0
      KVD=1
      DO 10 I=1,NSLB
      DO 20 J=2,NXC(I)
C------------------------------------------------------------------
C  DETERMINE ELEMENT SIDE DIMENSIONS A AND B
C------------------------------------------------------------------
         TA=XCRD(I,J)-XCRD(I,J-1)
         A=TA/2.0
         DO 30 K=2,NYC
            TBB=YCRD(K)-YCRD(K-1)
            B=TBB/(2.0*SIN(ANG))
            KEL=KEL+1
            CALL ZERO(F,8,1)
C------------------------------------------------------------------
C  PERFORM GAUSS POINT INTEGRATION
C------------------------------------------------------------------
            DO 40 IG=1,2
            DO 40 JG=1,2
             CALL BMFUN(BM,RG(IG),RG(JG),A,B,ANGLE,ANG)
```

```
            SCA=A*B*SIN(ANG)
            CALL SMULT(SCA,BM,3,8)
            CALL TRANMU(BM,ELAS,C,8,3,3)
            CALL MULT(C,THGR,SUM,8,3,1)
            CALL ADD(F,SUM,F,8,1)
 40        CONTINUE
C-----------------------------------------------------
C  CHECK FOR ELEMENTS WITH VOIDS UNDERNEATH
C-----------------------------------------------------
           IF (NVD .EQ. 0) THEN
             GOTO 60
           ELSE
             IF(KEL .NE. KVOID(KVD)) THEN
               GOTO 60
             ELSE
               KVD=KVD+1
               CALL ZERO(FTRACT,8,1)
               GOTO 70
             END IF
           END IF
C----------------------------------------------
C  DETERMINE WHICH QUADRANTS NODES ARE IN FOR
C  DIRECTION OF FRICTION
C----------------------------------------------
 60        CHK(1)=XMID(I)-XCRD(I,J-1)
           CHK(2)=XMID(I)-XCRD(I,J)
           CHK(3)=YMID-YCRD(K-1)
           CHK(4)=YMID-YCRD(K)
C-------------------------------------------------------
C  CALL SUBROUTINE TO FORM FRICTION FORCES FOR ELEMENT
C-------------------------------------------------------
cc           CALL TRACLD(A,B,FTRACT,KEL,CHK,2,ANG)
cc           CALL SMULT(FRACT,FTRACT,8,1)
C------------------------------------------------------
C  INSERT ELEMENT FORCES INTO STRUCTURAL LOADING ARRAY
C------------------------------------------------------
 70        DO 45 LK=1,4
             LLK=INDEX(KEL,LK)
             MMK=IDOF(LLK,1)
             NNK=IDOF(LLK,2)
             KKV(2*LK-1)=MMK
             KKV(2*LK)=NNK
 45        CONTINUE
              DO 50 L=1,8
             IF(KKV(L) .EQ. 0) GOTO 50
             KID=KKV(L)
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cc            IF(DABS(FTRACT(L,1)) .GT. DABS(F(L,1))) THEN
cc              SF=FTRACT(L,1)/DABS(FTRACT(L,1))
cc              FTRACT(L,1)=SF*DABS(F(L,1))
cc            END IF
cc            PLOAD(KID,1)=PLOAD(KID,1)+F(L,1)+FTRACT(L,1)
```

```
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
               PLOAD(KID,1)=PLOAD(KID,1)+F(L,1)
C
 50         CONTINUE
 30       CONTINUE
 20    CONTINUE
 10    CONTINUE
 510   FORMAT(I4,2X,E15.7,3X,I4)
       RETURN
       END
C
C-----------------------------------
C      SUBROUTINE TRACLD
C-----------------------------------
C
       SUBROUTINE TRACLD(A,B,FTRACT,KEL,CHK,KEY,ANG)
       IMPLICIT REAL*8 (A-H,O-Z)
CC       INTEGER TNEQ
       INTEGER TNODE
       DIMENSION QB(12,1),STRACT(8,12),CMT(8,2),RB(2,12),
      +TRACT(8,12),CHK(4),RG4(4),WTF(4)
       COMMON/GRID/XCRD(3,41),YMID,XMID(3),G1,G2,TNODE,
      +NSLB,NYC,NXC(3),IDOF(693,2),NTOTEL,KVOID(100),NVD
       COMMON/PLATE/QPL(2079,1)
       COMMON/PROP/EC,POI,ESHR,PHI,SGSTF
       COMMON/BLOC1/ELAS(3,3),RG(2)
C-----------------------------------------------------
C VARIABLE   TYPE     DESCRIPTION
C-----------------------------------------------------
C CMT    RM     MEMBRANE ELEMENT DISPLACEMENT FUNCTIONS
C RB     RM     PLATE ELEMENT DISPLACEMENT FUNCTIONS
C QB     RM     ARRAY OF NODAL PLATE DISPLACEMENTS
C RG4    RM     GAUSS POINTS FOR SIXTH ORDER INTEGRATION
C WTF     RM    WEIGHTING FACTORS FOR THIRD ORDER GAUSS
C TRACT      RM
C STRACT     RM         INTERMEDIATE STEPS IN INTEGRATION
C--------------------------------------------------------
C
C--------------------------------------------------------
C  EXTRACT PLATE DISP. FOR ELEMENT FROM STRUCTURAL ARRAY
C--------------------------------------------------------
       DO 10 I=1,4
          KQB=3*I
          KQPL=3*INDEX(KEL,I)
          QB(KQB-2,1)=QPL(KQPL-2,1)
          QB(KQB-1,1)=QPL(KQPL-1,1)
          QB(KQB,1)=QPL(KQPL,1)
 10    CONTINUE
C------------------------------------------------------
C  CALCULATE GAUSS POINTS AND WEIGHTING FACTORS
C------------------------------------------------------
         RG4(1)=-0.8611363116
```

```
        RG4(2)=-0.3399810436
        RG4(3)= 0.3399810436
        RG4(4)= 0.8611363116
C
        WTF(1)= 0.3478548451
        WTF(2)= 0.6521451549
        WTF(3)= 0.6521451549
        WTF(4)= 0.3478548451
C
      DO 20 I=1,10,3
         IF(QB(I,1) .LT. 0.0) QB(I,1)=0.0
 20   CONTINUE
        CALL ZERO(STRACT,8,12)
C-----------------------------------------------
C  PERFORM INTEGRATION FOR FRICTION FORCES
C-----------------------------------------------
      DO 30 IG=1,4
        DO 35 JG=1,4
           CALL CMTFUN(CMT,RG4(IG),RG4(JG))
           CALL RBFUN(RB,RG4(IG),RG4(JG),A,B)
           CALL MULT(CMT,RB,TRACT,8,2,12)
           SCA=WTF(IG)*WTF(JG)
           CALL SMULT(SCA,TRACT,8,12)
           CALL ADD(STRACT,TRACT,STRACT,8,12)
 35     CONTINUE
 30   CONTINUE
       SCA=A*B*PHI*SGSTF*SIN(ANG)
      CALL SMULT(SCA,STRACT,8,12)
      CALL MULT(STRACT,QB,FTRACT,8,12,1)
C-----------------------------------------------------
C  ADJUST FORCES FOR QUADRANT AND BOUNDARY CONDITIONS
C-----------------------------------------------------
        IF(DABS (CHK(1)) .LT. 0.001 ) THEN
           FTRACT(1,1)=0.0
           FTRACT(3,1)=0.0
        ELSE
           IF( CHK(1) .GT. 0.0) THEN
              FTRACT(1,1)=-DABS(FTRACT(1,1))
              FTRACT(3,1)=-DABS(FTRACT(3,1))
           ELSE
              FTRACT(1,1)=DABS(FTRACT(1,1))
              FTRACT(3,1)=DABS(FTRACT(3,1))
           END IF
        END IF
        IF(DABS (CHK(2)) .LT. 0.001 ) THEN
           FTRACT(5,1)=0.0
           FTRACT(7,1)=0.0
        ELSE
           IF( CHK(2) .GT. 0.0) THEN
              FTRACT(5,1)=-DABS(FTRACT(5,1))
              FTRACT(7,1)=-DABS(FTRACT(7,1))
           ELSE
```

```
            FTRACT(5,1)=DABS(FTRACT(5,1))
            FTRACT(7,1)=DABS(FTRACT(7,1))
         END IF
      END IF
      IF(DABS (CHK(3)) .LT. 0.001 ) THEN
         FTRACT(2,1)=0.0
         FTRACT(6,1)=0.0
      ELSE
         IF( CHK(3) .LT. 0.0) THEN
            FTRACT(2,1)= DABS(FTRACT(2,1))
            FTRACT(6,1)=DABS(FTRACT(6,1))
         ELSE
            FTRACT(2,1)=-DABS(FTRACT(2,1))
            FTRACT(6,1)=-DABS(FTRACT(6,1))
         END IF
      END IF
      IF(DABS (CHK(4)) .LT. 0.001 ) THEN
         FTRACT(4,1)=0.0
         FTRACT(8,1)=0.0
      ELSE
         IF( CHK(4) .LT. 0.0) THEN
            FTRACT(4,1)= DABS(FTRACT(4,1))
            FTRACT(8,1)=DABS(FTRACT(8,1))
         ELSE
            FTRACT(4,1)=-DABS(FTRACT(4,1))
            FTRACT(8,1)=-DABS(FTRACT(8,1))
         END IF
      END IF
C-----------------------------------------------------
C  REVERSE SIGNS OF FORCES FOR THERMAL EXPANSION
C-----------------------------------------------------
      IF( KEY .EQ. 2) THEN
        SC=-1.0
        CALL SMULT(SC,FTRACT,8,1)
      END IF
      RETURN
      END
C
C
C
      SUBROUTINE TRANMU(A,B,C,L,M,N)
C-----------------------------------------------------
C  CALCULATE [A] TRANSPOSE X [B] = [C] WHERE A IS
C  DIMENSIONED M X L, B IS M X N, AND C IS L X N
C-----------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(M,L),B(M,N),C(L,N)
      DO 10 I=1,L
      DO 10 J=1,N
      C(I,J)=0.0
        DO 20 K=1,M
        C(I,J)=C(I,J)+A(K,I)*B(K,J)
```

```
 20      CONTINUE
 10    CONTINUE
       RETURN
       END
C
       SUBROUTINE ZERO(A,N,M)
C---SUBROUTINE TO ZERO OUT AN N X M MATRIX A
       DOUBLE PRECISION A
       DIMENSION A(N,M)
       DO 10 I=1,M
       DO 10 J=1,N
 10    A(J,I)=0.0
       RETURN
       END
```

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BIOGRAPHICAL SKETCH

Byung-Wan Jo was born on September 20, 1956, in Cheong-Ju city, Korea. He grew up there and graduated from Cheong-Ju High School in February, 1975. He entered the Han-Yang University, Seoul, Korea, and received his Bachlor of Science degree in civil engineering in February, 1979.

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In August, 1985, he received his Master of Science degree in civil engineering of the Ohio University, U.S.A. and started the Ph.D program at the Universty of Florida in Gainsville, U.S.A., where he expects to graduate in April, 1988.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Associate Professor of Civil Engineering

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Mang Tia

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